When Should Sellers Use Auctions?

Contains color figures.

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Abstract

When it is costly to participate in the sale of an asset, potential buyers typically submit offers either simultaneously, as in an auction, or sequentially, so that later potential entrants may observe previous bids. We show in an IPV setting that when the entry process is selective, so that buyers with higher values are more likely to enter, both sellers and buyers generally prefer the sequential process and that the difference in seller revenues can be large. We illustrate our findings with parameters estimated from simultaneous entry, open outcry USFS timber auctions. We predict that using the sequential mechanism would raise the USFS's revenues by 5% in a representative auction, which is many times larger than the gain to setting an optimal reserve price.

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1 Introduction

Sellers typically engage buyers in one of two ways when selling an asset. Buyers either compete (i) *simultaneously*, as they would in a typical auction, or (ii) *sequentially*, whereby the seller agrees to a price with one buyer while retaining the right to approach others in search of a better offer.[1] There is a great deal of literature showing that when participation in the sales process is costless, a seller should prefer simultaneous competition due to auctions' desirable efficiency and revenue maximization properties. However, participation is rarely costless. For example, firms often need to undertake costly investment to more precisely learn their value for an asset prior to purchase. Moreover, if participation is costly, the seller's choice of mechanism is far from certain. For although a sequential mechanism that allows later potential entrants to use information from previous bidders to inform their participation decisions will be more efficient than an auction, this in and of itself does not mean that the seller should prefer it to an auction. The relevant question to him is whether he can capture the increased rents.

In this paper we show that a seller should typically prefer a sequential mechanism to an auction and illustrate our findings by showing empirically in a relevant setting that the US Forest Service (USFS) could substantially improve their revenues if they switched their sales mechanism from a simultaneously competitive open outcry auction to a sequential sales process. To our knowledge this paper is the first structural analysis and empirical comparison of these two methods.

At the heart of our work is a flexible model of entry where potential bidders, in either the auction or the sequential mechanism, can be asymmetric and may be more likely to enter if they value the good more. In this sense the entry model can imperfectly *selective*.[2] Otherwise, we maintain the assumptions of the standard auction model (single good, independent private values, no quality, credit or renegotiation concerns). In our two-stage entry model, each firm receives a private information, noisy signal about its value before taking a costly entry decision, after which the firm learns its true value. A firm's equilibrium entry strategy involves entering if its signal is high enough, so firms with higher values will be more likely to enter. The precision of the signal determines how selective the entry process is. In the limits, the model can approach the polar cases of (a) perfect selection, whereby a firm knows its value exactly when taking its entry decision, and (b) no selection, whereby a firm knows nothing of its value when taking its entry decision. Most of the literature focuses on these two polar cases. We view moderately selective entry as a plausible description of many auction settings (such as the sale of timber or oil and gas leases, government procurement contracts, firm takeovers) where all potential buyers are likely to have some idea of

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[1]Oftentimes the sequential process is operationalized through a "go-shop" clause whereby a seller comes to an agreement on an initial price with a buyer but retains the right to solicit bids from other buyers for the next 30-60 days. If a new, higher offer is received, then according to the "match right", which is often included in the agreement with the initial buyer, the seller must negotiate with the first buyer (for 3-5 days, for example) to see if it can match the terms of the new, higher offer. See Subramanian (2008) for a recent analysis of the increased use of "go-shop" clauses in the M&A market.

[2]Selective entry contrasts with standard assumptions in the empirical entry literature (e.g., Berry (1992)) where entrants may differ from non-entering potential entrants in their fixed costs or entry costs, but not in characteristics such as marginal costs or product quality that affect their competitiveness or the profits of other firms once they enter.

how they value the product, but must undertake research in order to learn their exact value.

We use this entry model to compare the two most common methods of selling an asset. The first process we consider is a simultaneous entry second price auction. The second is a similarly simple sequential sales process. In this second mechanism, potential buyers are approached in turn. If a potential buyer chooses to participate in the mechanism, he names the price at which he would buy the item, knowing that the seller retains the right to approach other buyers in hopes of increasing the sales price. Should the seller find another buyer who, upon observing previous offers for the good, decides to participate, the two active firms bid against themselves to establish the (possibly new) incumbent buyer. The incumbent at the end of the game pays the standing price. As noted elsewhere (Wasserstein (2000) and Bulow and Klemperer (2009)), these alternatives can be thought of as spanning the range of possible sales mechanisms.

We show that for a wide range of plausible parameters, the sequential mechanism gives the seller higher expected revenues and the differences in expected revenues can be quite large. We show how the degree of selection, the level of entry costs and the degree to which bidders are asymmetric affect the relative performance of the two mechanisms. There are at least two fundamental reasons why selective entry tends to lead to the sequential mechanism performing better. First, with no selection, entry ceases in the sequential mechanism once a firm enters that has a high enough value, which it can signal by posting a deterring bid, so that later entrants will not enter even if they have high values. In contrast, with selection, there is always some probability that a later firm with a value above the incumbent will enter. This is true because (i) a potential entrant may get a very favorable signal about its value; and (ii) in the equilibrium we consider, which is unique under standard refinements, (almost all) incumbents bid in a way which perfectly reveals their value to later competitors. These features make the sequential mechanism particularly efficient when there is selection, and the seller only has to capture some of this additional surplus to be better off. Second, firms may have to bid more aggressively to maintain the separating equilibrium. In contrast, when there is no selection, the unique sequential equilibrium of the model under standard refinements involves all incumbents with values above some threshold submitting the same deterring bid (pooling), and this deterring bid may be relatively low. This last result underlies the findings in Bulow and Klemperer (2009) (BK hereafter) who show that in a setting with no selection, low entry costs and symmetric bidders a seller should usually prefer an auction. We also note, however, that not only can the addition of selection overturn this result, but even in a setting with no selection, if entry costs are high or buyers are asymmetric, the seller may prefer the sequential process to the auction.

In our empirical investigation we find a moderate amount of selection in the decision to participate in the USFS's open outcry timber auctions. This stems from potential buyers' (who are sawmills or loggers), need to perform "cruises" of timber tracts to accurately establish their values.[3] Using our structural estimates we predict the USFS could increase their expected revenues in a representative auction by 4.8% by using the sequential mechanism rather than the auction format

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[3]Different potential buyers may value different kinds of species differently and some of the details which the cruises may establish include the volumes of each species, as well as the likely costs of clearing the tract.

that was actually used. Moreover, this revenue difference is much larger than the gains to setting optimal reserve prices in the auction, which so far has been the key policy tool considered.[4] In the representative auction, implementing the optimal reserve improves revenues only 0.2% relative to using no reserve. Of course, it may be that there are benefits to using simultaneous-entry auctions, such as transparency, which we do not consider. However, our results suggest that these benefits must be quite large to rationalize current practice.

Three comments about the nature of our results are appropriate. First, our goal is not to find the seller's optimal mechanism. Instead, in the same spirit as BK, we want to compare mechanisms that sellers typically use to illustrate when potential future competition can discipline buyer behavior in such a way that the seller benefits more than if the buyers competed simultaneously. Moreover, we seek to provide the first (to our knowledge) counterfactual estimates of a seller's return from switching from a simultaneous entry auction to a sequential mechanism in a relevant empirical setting. In fact, in a setting with endogenous, partially selective entry with asymmetric bidders, the optimal mechanism is not even known. This being said, results from the optimal mechanism design literature for the extreme cases where buyers either receive no signals before entering or know their values for sure before paying the entry cost, suggest that the optimal mechanism would be sequential, even if it would also have some less plausible features such as additional entry fees and bid-dependent payments to all firms that decide to enter.[5] These features seem unlikely to be implemented in practice and setting them appropriately would also require the seller to have accurate knowledge of the parameters of the model. In contrast, the sequential mechanism considered here imposes the same informational demands on the seller as running the auction, except that an exhaustive set of potential buyers must be identified. One can therefore view our empirical results as providing a lower bound on how much better the USFS, or similar sellers, could do by using well-designed sequential mechanisms.

Second, while we characterize the equilibria we consider for each mechanism and show that they are unique under refinements, our revenue comparisons are computational rather than analytical. This reflects the fact that we are relaxing two assumptions (no selection and symmetry) that make algebraic analysis tractable. We view one contribution of our paper as showing that simplifications that have been made for tractability may give misleading impressions about how well auctions actually work.

Third, we recognize that we are not the first to call the performance of standard auctions into question in empirically important settings. However, auctions' superiority has typically been

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[4]Some examples of previous studies of timber auction reserve prices include Mead, Schniepp, and Watson (1981), Mead, Schniepp, and Watson (1984), Paarsch (1997), Haile and Tamer (2003), Li and Perrigne (2003) and Aradillas-Lopez, Gandhi, and Quint (2010b). All of these papers assume that entry is not endogenous.

[5]Cremer, Spiegel, and Zheng (2009) characterize the mechanism design problem when uninformed potential buyers pay the entry cost. In that case, the sequential mechanism implements a sequential search rule where buyers are engaged in turn, with entry fees being charged and various reimbursements being made to some bidders. McAfee and McMillan (1988) characterize the design problem when potential buyers know their values but there is a cost to engaging each additional potential buyer. The optimal mechanism once again implements a sequential search amongst buyers, which ceases once a buyer with a high enough value is found. Finally, Manelli and Vincent (1995) study whether an auction is optimal when sellers care about quality as well as price.

challenged only in settings where additional concerns such as item complexity (Asker and Cantillon (2008)), budget constraints (Che and Gale (1996)), bankruptcy (Zheng (2001)) or renegotiation (Bajari, McMillan, and Tadelis (2009)) are important. In contrast, we focus on the entry margin, which is relevant in almost all settings.

The paper proceeds as follows. Section 2 introduces the models of each mechanism and characterizes the equilibria that we examine. Section 3 compares expected revenues from the two mechanisms for wide ranges of parameters, and provides some intuitions for when the sequential mechanism performs better than the auction. Section 4 describes the empirical setting of USFS timber auctions and explains how we estimate our model. Section 5 presents the parameter estimates and some initial comparisons of the revenue performance of each mechanism. Section 6 concludes.

2 Model

We now describe our model of firms' values and their signals, before describing the mechanisms that we are going to compare and their associated equilibria.

2.1 A General Entry Model with Selection

Suppose that a seller has one unit of a good to sell, and that the seller gets a payoff of zero if the good is unsold. There is a set of potential buyers who may be one of $\tau = 1, ..., \overline{\tau}$ types, with N_τ of type τ. Buyers have independent private values (IPV) which can lie on $[0, \overline{V}]$, distributed according to $F_\tau^V(V)$. F_τ^V is continuous and differentiable for all types. In this paper we will typically assume that the density of V is proportional to the log-normal distribution on $[0, \overline{V}]$, and that $\overline{V}$ is high, so that the density of values at $\overline{V}$ is very small.[6] Before participating in any mechanism, a potential buyer must pay an entry cost K_τ (and we assume that a firm cannot choose not to pay K.) Once it pays K_τ, a potential buyer learns its value. However, prior to deciding whether to enter, a bidder receives a private information signal about his value. We focus on the case where the signal of potential buyer i of type τ is determined by

$$s_{i\tau} = v_{i\tau} a_{i\tau} \text{ where } A_\tau = e^{\varepsilon_\tau}, \varepsilon_\tau \sim N(0, \sigma_{\varepsilon\tau}^2)$$

In this model the variance of the εs controls how much potential buyers know about their values before deciding whether to enter. As $\sigma_{\varepsilon\tau}^2 \to \infty$, the model will tend towards the informational assumptions of the Levin and Smith (1994) model (signals are uninformative, LS model), while as $\sigma_{\varepsilon\tau}^2 \to 0$ it tends towards the informational assumptions of the Samuelson (1985) (S) model where firms know their values prior to paying an entry cost (which is therefore interpreted as a bid preparation or attendance cost). As buyers with higher values are more likely to be allocated the good in both of the mechanisms that we consider, entry will be less selective as σ_ε^2 increases. In general, intermediate values of $\sigma_{\varepsilon\tau}^2$, implying that buyers have some idea of their values but have to

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[6]To be precise, $f_\tau^V(v|\theta) = \frac{g(v|\theta)}{\int_0^{\overline{V}} g(x|\theta)dx}$ where $g(v|\theta)$ is the pdf of the log-normal distribution.

conduct costly research to learn them for sure, seem plausible for most empirical settings. Draws of ε are i.i.d. across bidders, and, having received his signal, a potential buyer forms posterior beliefs about his valuation using Bayes Rule.

We note that there are three differences between this model and the model considered by BK. First, we assume that potential entrants get some type of signal about their value prior to entering whereas BK assume that potential buyers only know the common distribution of values (the Levin and Smith assumption). Second, we allow for asymmetries between buyer types. Both of these changes require us to use computational techniques to compare revenues across mechanisms. Third, we assume that there is a fixed and known set of potential entrants which could, of course, differ across auctions. BK's model is more general allowing for some probability ($0 \leq \rho_j \leq 1$) of a jth potential entrant if there are $j-1$ potential entrants.

2.2 Mechanism 1: Simultaneous Entry Second Price Auction

The first mechanism we consider is a simultaneous entry second price or open outcry auction. The auctions that we observe in the data will have an open outcry format. We note that this is slightly different from the main auction model which BK study, which has sequential entry (but firms place bids simultaneously). However, as BK argue, a simultaneous entry auction model would give similar results, and in general the simultaneous entry model seems a more reasonable description of most auctions in the real world.[7] In this mechanism all potential buyers first simultaneously decide whether to enter (pay K_τ) the auction based on their signal, which is private information, the number of potential entrants of each type and the auction reserve price, which are assumed to be common knowledge to all potential buyers. Entrants then learn their values and submit bids. We assume that an open outcry auction would give the same outcome as an English button auction, so that the good would be awarded to the firm with the highest value at the price which would make the firm with the second-highest value drop out of the auction or the reserve price if the seller uses a reserve.[8]

Following the literature (e.g. Athey, Levin, and Seira (forthcoming)), we assume that players use strategies that form type-symmetric Bayesian Nash equilibria, where "type-symmetric" means that every player of the same type will use the same strategy. Consider any auction a. In the second stage, entrants know their values so it is a dominant strategy for each entrant to bid its value. In the first stage, players take entry decisions based on what they believe about their value given their signal. By Bayes Rule, the (posterior) conditional density $g_{\tau a}(v|s_i)$ that a player of type τ's value

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[7] "No important result is affected if potential bidders make simultaneous, instead of sequential, entry decisions into the auction" (page 1560).

[8] When we introduce our estimation strategy in Section 4, we extend our methodology to cover more general models of bidding in open outcry auctions which does not require all bidders to bid up to their true value.

is v when its signal is s_i is:

$$g_{\tau a}(v|s_i) = \frac{f^V_{\tau a}(v) \times \frac{1}{\sigma_{\varepsilon a}}\phi\left(\frac{\ln\left(\frac{s_i}{v}\right)}{\sigma_{\varepsilon a}}\right)}{\int_0^{\overline{V}} f^V_{\tau a}(x) \times \frac{1}{\sigma_{\varepsilon a}}\phi\left(\frac{\ln\left(\frac{s_i}{x}\right)}{\sigma_{\varepsilon a}}\right)dx} \tag{1}$$

where $\phi\left(\cdot\right)$ denotes the standard normal pdf.

The weights that a player places on its prior and its signal when updating its beliefs about its true value depend on the relative variances of the distribution of values and ε (signal noise), and this will also control the degree of selection. A natural measure of the relative variances is $\frac{\sigma^2_{\varepsilon a}}{\sigma^2_{Va}+\sigma^2_{\varepsilon a}}$, which we will denote α_a. If the value distribution were not truncated above, a player i's (posterior) conditional value distribution would be lognormal with location parameter $\alpha_a\mu_{\tau a} + (1 - \alpha_a)s_i$ and squared scale parameter $\alpha_a\sigma^2_{V\tau a}$.

The optimal entry strategy in a type-symmetric equilibrium is a pure-strategy threshold rule whereby the firm enters if and only if its signal is above a cutoff, $S'^*_{\tau a}$.[9] $S'^*_{\tau a}$ is implicitly defined by the zero-profit condition that the expected profit from entering the auction of a firm with the threshold signal will be equal to the entry cost:

$$\int_{R_a}^{\overline{V}}\left[\int_{R_a}^v (v-x)h_{\tau a}(x|S'^*_{\tau a}, S'^*_{-\tau a})dx\right]g_{\tau a}(v|s)dv - K_a = 0 \tag{2}$$

where $g_{\tau a}(v|s)$ is defined above, and $h_{\tau a}(x|S'^*_{\tau a}, S'^*_{-\tau a})$ is the pdf of the highest value of other entering firms (or the reserve price R_a if no value is higher than the reserve) in the auction.

A pure strategy type-symmetric Bayesian Nash equilibrium exists because optimal entry thresholds for each type are continuous and decreasing in the threshold of the other type.

With multiple types, there can be multiple equilibria in the entry game even when we assume that only " type-symmetric" equilibria (i.e., ones in which all firms of a particular type using the same strategy) are played. As explained in Roberts and Sweeting (2011), we choose to focus on an equilibrium where the type with higher mean values has a lower entry threshold (lower thresholds make entry more likely). This type of equilibrium is intuitively appealing and when firms' reaction functions are S-shaped (reflecting, for example, a normal or log-normal value distribution) and types only differ in the location parameters of their value distributions (i.e., the scale parameter, signal noise variance and entry costs are the same) then there is a unique equilibrium of this form.[10]

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[9]A firm's expected profit from entering is increasing in its value, and because values and signals are independent across bidders and a firm's beliefs about its value is increasing in its signal, a firm's expected profit from entering is increasing in its signal. Therefore, if a firm expects the profit from entering to be greater (less) than the entry cost for some signal S, it will also do so for any signal $\widetilde{S}$ where $\widetilde{S} > S$ ($\widetilde{S} < S$). As it will be optimal to enter when the firm expects the profit from entering to be greater than the entry costs, the equilibrium entry strategy must involve a threshold rule for the signal, with entry if $S > S'^*_{\tau a}$.

[10]As we mention in Section 4, we have estimated the model using a nested pseudo-likelihood procedure which does not require us to use an equilibrium selection rule. The parameter estimates in this case indicate that the difference in mean values between our two types (sawmills and logging companies) are so large that multiple equilibria cannot be supported.

2.3 Mechanism 2: Sequential Mechanism of Bulow and Klemperer (2009)

The alternative mechanism we consider is similar to BK's sequential mechanism, which they suggest reflects an alternative to standard auctions that approximately describes what happens in practice in settings such as the sale of a company.

The mechanism operates in the following way. Potential buyers are placed in some order (which does not depend on their signals, but may depend on types), and the seller approaches each potential buyer in turn. We will call what happens between the seller's approach to one potential buyer and its approach to the next potential buyer a "round". In the first round, the first potential buyer observes his signal and then decides whether to enter the mechanism and learn his value by paying K_τ. If he enters he can choose to place a 'jump bid' b_1 above the reserve price, which we assume to be zero. Given entry, submitting a bid is costless.

In the second round the potential buyer observes his signal, the entry decision of the first buyer and his jump bid, and then decides whether to enter himself. If the first firm did not enter and the second firm does, then the second firm can place a bid in exactly the same way as the first firm would have been able to do had he entered. If both enter, the firms bid against each other in a knockout button auction until one firm drops out, in which case the drop-out firm can never return to the mechanism. The remaining firm then has an opportunity to submit an additional higher jump bid above the bid at which the other firm dropped out. If the second firm does not enter, but the first firm did, then the first firm can either keep its initial bid or submit a higher jump bid.

This procedure is then repeated for each remaining potential buyer, so that in each round there is at most one incumbent bidder coming from the previous round and one potential entrant. The complete history of the game (entry decisions and bids, but not signals) is observed by all players. The history up to round n will be denoted Γ_n. If a firm drops out, or chooses not enter, it is assumed to be unable to re-enter at a later date. At the end of the game the good is allocated to the remaining bidder at a price equal to the current bid.

A strategy in the sequential model will consist of an entry rule and a bidding rule as a function of the round, the potential buyer's signal and value (for bidding) and the observed history. When a potential buyer is bidding against an active opponent the dominant strategy is to bid up to its value, so that the firm with the lower value will drop out at a price equal to its value. This does not depend on the fact that we have a selective entry model because values are known at that stage. However, the strategies that firms use to determine their jump bids and entry decisions do depend on selective entry. To place our equilibrium in context, we begin describing what happens when there are no signals and symmetric firms, which are the assumptions made by BK.

2.3.1 Equilibrium with No Pre-Entry Signals

BK show that there is a unique perfect sequential equilibrium in entry and jump bidding strategies where in all rounds before the last round an incumbent will, depending on its value, either submit no jump bid (i.e., keep its bid equal to its level from the previous round or the level at which it defeated the other firm in the current round) or will submit a jump bid which, in equilibrium, deters

all future entry. Assuming only one type of firm, all firms with values below some V^S will keep the existing standing bid b', while all firms with values above V^S will submit a jump bid equal to b''. V^S will be determined by the condition that future potential entrants should be indifferent to entering when the incumbent firm's value is above V^S

$$\int_{V^S}^{\overline{V}} \int_{V^S}^{\overline{V}} (x - v') f^V(v') f^V(x) dv' dx - K = 0$$

where this condition recognizes the fact that an entrant will only win if its value is above the incumbent and that in this case it will defeat the incumbent at a price equal to its value and then deter entry in all future periods. In the equilibrium BK consider, V^S will be the same independent of the round the game is in, or the history of the game to that date. The deterring bid b'' is determined by the condition that the bidder with value V^S should be indifferent between deterring future entry with a bid of b'' (which it will pay because it will win) and allowing entry to occur by keeping the standing bid b' (it may not win, but if it does it will pay a lower price than b''). b'' may depend on b' (which will depend on the history of the game) and the number of rounds remaining.

Equilibrium outcomes with no signals are therefore characterized by entry ceasing completely as soon as there is an entrant who has a high enough value. As BK show, this leads to a more socially efficient outcome than an auction, because entry only occurs when the probability that an entrant will raise the total surplus is relatively large. However, because deterring bids may be relatively low and more entry would increase prices, this equilibrium generally produces lower revenues for sellers than the auction.

2.3.2 Equilibrium with Pre-Entry Signals

When potential buyers receive pre-entry signals, there are important changes to the nature of the equilibrium. We begin by describing the equilibrium we consider, before explaining the refinements which lead us to focus on it. We also provide examples showing how equilibrium strategies affect outcomes and how the parameters affect equilibrium strategies.

The equilibrium we consider can be characterized as follows:

- A potential entrant enters if its signal is above some threshold, which will depend on the round and its beliefs about the value of the incumbent if there is one. For example, the final potential entrant (round N) will enter if and only his signal is above $S_N'^*$ where $S_N'^*$ is defined by the following zero profit condition:

$$\int_{b}^{\overline{V}} \int_{x}^{\overline{V}} (x - v') h(v'|\Gamma_N) g^V(x|S_N'^*) dv' dx - K = 0$$

where $h(v'|\Gamma_N)$ is the density describing the potential entrant's belief about the incumbent's value given the history of the game, $g^V(x|S_N'^*)$ is the potential entrant's own posterior belief about its value given its signal and b is the standing bid. When $\overline{V}$ is high, there will be some

probability of entry for almost all beliefs about the incumbent's value. A similar condition, with additional notation to reflect a firm's expectations about whether future entry will happen and how it might affect the price paid, will define $S_n'^*$ for earlier rounds ($n < N$). If a firm believes that the incumbent's value is certainly more than $\overline{V} - K$ then it will not enter;

- Any firm will bid up to its value in a knockout auction (this is a dominant strategy, just as in the game with no signals prior to entry);

- Incumbents with values above the standing bid will place a jump bid at their first opportunity to do so. For values less than $\overline{V} - K$, the jump bid will perfectly reveal the incumbent's valuation, i.e., there will be a fully separating equilibrium for values less than $\overline{V} - K$. The equilibrium bid function in this case will be determined by a first-order differential equation and the initial condition that a firm with value equal to the standing bid will keep the standing bid. As an example, consider the decision of a new incumbent in the penultimate round. Given a bid function $b(v)$, which will reveal its value to the potential entrant, the incumbent has to decide which v's bid he should submit:

$$
\max_{v'} \int_0^{\overline{V}} \int_0^{S_N'^*(v')} (v - b(v'))q(s|x)f^V(x)dsdx + \int_0^{b(v')} \int_{S_N'^*(v')}^{\infty} (v - b(v'))q(s|x)f^V(x)dsdx
$$

$$
+ \int_{b(v')}^{v} \int_{S_N'^*(v')}^{\infty} (v - x)q(s|x)f^V(x)dsdx \tag{3}
$$

where $S_N'^*(v')$ is the final potential entrant's threshold for entry when it believes the incumbent has exact value v' and $q(s|x)$ is the density of the potential entrant's signal when its value is x. The first two terms reflect the incumbent's expected profit when it keeps the final potential entrant out or the entrant comes in but has a value less than the standing bid, so that the incumbent pays its bid. The third term reflects the incumbent's expected profit when the final potential entrant enters and has a value above $b(v')$. Differentiating this objective function with respect to v', and requiring that the first order condition is equal to zero when $v = v'$ (so that local incentive compatibility constraints are satisfied), gives the differential equation that defines the bid function. The lower boundary condition is provided by $b(b') = b$, i.e., incumbents with values less than or equal to the standing bid will submit the standing bid. In equilibrium, the lowest value an incumbent will ever have is the standing bid, because no firm should ever bid above its value. In the equilibrium we consider, incumbents only choose to submit jump bids once: these bids reveal their value to all future players, so that in later rounds they do not raise the standing bid. The bidding problem in earlier rounds can be defined in a similar way with additional notation. Firms with values above $\overline{V} - K$ will pool, submitting bids equal to $b(\overline{V} - K)$. For high $\overline{V}$, this will happen very rarely;

- When an incumbent submits a jump bid b less than $b(\overline{V} - K)$, a potential entrant's posterior belief about the incumbent's value will place all of the weight on $b^{-1}(b)$. For bids at $b(\overline{V}-K)$, the entrant's beliefs will be consistent with Bayes Rule.

2.3.3 Equilibrium Refinement

So far we have described one equilibrium of the sequential model, where jump bids are fully revealing up to $\overline{V} - K$, but there may be other equilibria. However, given our assumptions, we can show that our equilibrium is the only one consistent with the " D1 Refinement" (Banks and Sobel (1987), Cho and Kreps (1987)), which has been widely used in the theoretical literature on signaling models (Fudenberg and Tirole (1991)). To make our arguments as clear as possible, we consider the case with two potential entrants, which matches existing models in the signalling literature quite closely. We then consider the extension to the case with more firms. As the distribution of values has the same support for all types, adding more types has no effect on our arguments, so we assume there is only one type to reduce notation.

With two rounds, the equilibrium has the following form when the first firm enters and has a value on $[0, \overline{V} - K]$: (1) the jump bid of an incumbent (first round entrant), $b_1^*(v_1)$, is a smooth, monotonically increasing and differentiable function in the firm's value; (2) a second round potential entrant seeing b_1 places probability 1 on the first round entrant having value $b_1^{*-1}(b_1)$; (3) the second round potential entrant enters if he expects positive surplus given this belief about 1's value and the signal about his own value; (4) the bid function is defined by solving the differential equation implied by the first-order condition requiring each incumbent being willing to submit the bid associated with his true value rather than locally deviating; and (5) a boundary condition where the lowest value incumbent submits the lowest possible bid (zero).

The refinement will depend on three properties of the game. Let $\pi_v(b_1, S_2'^*)$ be the expected profit of a first round incumbent, where v is the incumbent's value, b_1 is his bid and $S_2'^*$ is the entry threshold of the second round potential entrant (which will be a function of b_1). Given our assumptions and the dominant strategies in the knockout game and in the absence of entry, $\pi_v(b_1, S_2'^*)$ will be continuous and differentiable in both its arguments. The three properties are:

1. $\frac{\pi_v(b_1, S_2'^*)}{\partial S_2'^*} > 0$;

2. $\frac{\pi_v(b_1, S_2'^*)}{\partial b_1} / \frac{\pi_v(b_1, S_2'^*)}{\partial S_2'^*}$ is monotonic in v;

3. $S_2'^*$ is uniquely defined for any belief about the first potential entrant's value, and $S_2'^*$ is increasing.

We verify that these properties hold in the Appendix.

Mailath (1987) shows that an equilibrium bid function can be found using the differential equation (i.e., only checking local deviations) and the boundary condition in a continuous type signaling model when the single crossing condition holds. Mailath (1987)'s results also imply that our equilibrium will be the unique *separating* sequential equilibrium, although his results do not rule out the existence of a pooling equilibrium on the $[0, \overline{V} - K]$ interval. However, Ramey (1996) (which extends the results in Cho and Sobel (1990) to the case of an unbounded action space and continuous types on an interval) shows that the three properties outlined above imply that only a separating equilibrium will satisfy the D1 refinement, so our equilibrium must be the only sequential equilibrium

satisfying D1. As noted by Mailath (1987), this equilibrium will also be the separating equilibrium which is least costly to the first round potential entrant. This is helpful for us, because it implies that other equilibria would give even higher revenues to the seller.

The conditions also imply that, if an incumbent with value $\overline{V} - K$ prefers $b_1(\overline{V} - K)$, which will stop all future entry, to a lower bid then all incumbents with values above $\overline{V} - K$ will prefer $b_1(\overline{V} - K)$ to lower bids. But, firms with values above $\overline{V} - K$ will also strictly prefer to bid $b_1(\overline{V} - K)$ than any higher bid (for any beliefs of the potential entrant following a higher bid), because by bidding $b_1(\overline{V} - K)$ the incumbent can get the asset for sure at a lower price.

Three (or More) Rounds We now consider a model with three potential entrants (arguments for more rounds would follow directly from this case). We make a small simplification by restricting ourselves to equilibria where all potential entrants make the same inferences from a bid by an incumbent and incumbents only make jump bids in the first round that they enter.[11] The two period equilibrium discussed above would define strategies for the final two rounds if the second period entrant enters and defeats any incumbent entrant from the first round (with an adjusted boundary condition to reflect the new standing bid). It therefore only remains to argue that there is a unique sequential equilibrium bid function, which is fully separating for values $[0, \overline{V} - K]$, for a first round entrant. A first round entrant's jump bid sends a signal to the second round potential entrant, and, if he is still an incumbent in the final round, which must be the case if he is to win, the final potential entrant. Conditional on the incumbent surviving the second round, the third round is just a repeat of another two round game. The first round entrant's expected profit function is now $\pi_v(b_1, S_2'^*, S_3'^*)$, and the following properties hold:

1. $\frac{\partial \pi_v(b_1, S_2'^*, S_3'^*)}{\partial S_2'^*} > 0$ and $\frac{\partial \pi_v(b_1, S_2'^*, S_3'^*)}{\partial S_3'^*} > 0$;

2. $\frac{\partial \pi_v(b_1, S_2'^*, S_3'^*)}{\partial b_1} / \frac{\partial \pi_v(b_1, S_2'^*, S_3'^*)}{\partial S_2'^*}$ and $\frac{\partial \pi_v(b_1, S_2'^*, S_3'^*)}{\partial b_1} / \frac{\partial \pi_v(b_1, S_2'^*, S_3'^*)}{\partial S_3'^*}$ are each monotonic in v;

3. both $S_2'^*$ and $S_3'^*$ are uniquely defined for any belief about the first entrant's value, and they choose actions that are better for the first entrant when they believe that his value is higher.

These conditions allow us to apply the D1 refinement to the signaling game between the incumbent making the jump bid and every subsequent potential entrant.

2.3.4 Simple Examples Illustrating How the Mechanism Works

To provide some additional clarity about how the mechanism works, given equilibrium strategies, Table 1 presents what happens in two games with 4 potential entrants (rounds), and one type of firm with values distributed proportional to $LN(4.5, 0.2)$ on [0,200], $K = 4$ and $\sigma_\varepsilon = 0.2$.

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[11]It is possible that future potential entrants could ignore the information that they have on games before the last round. In this case, incumbents would choose to submit jump bids every round. This simplification allows us to consider a model where a firm sends at most one signal to many possible sequential receivers.

| | Initial | Potential Entrant | | | | Post-Knockout | Post-Jump Bid |
Round	Standing Bid	Value	Signal	s^*	Entry	Standing Bid	Standing Bid
				Example 1			
1	-	89.3	163.4	89.4	Yes	-	76.4
2	76.4	75.4	79.13	95.5	No	76.4	76.4
3	76.4	104.9	114.9	90.7	Yes	89.3	92.7
4	92.7	100.1	114.2	110.8	Yes	100.1	100.1
	Seller's Revenue 100.1, social surplus (winner's value less total entry costs) 92.9						
				Example 2			
1	-	71.3	54.6	89.4	No	-	-
2	-	109.3	137.7	83.9	Yes	0	92.2
3	92.2	99.1	122.5	121.0	Yes	99.1	99.1
4	99.1	117.8	93.0	119.4	No	99.1	99.1
	Seller's Revenue 99.1, social surplus (winner's value less total entry costs) 91.1						

Table 1: Simple examples of how the sequential mechanism works.

In both games, the first potential entrant enters if he receives a signal greater than 89.4. The signal thresholds in later rounds depend on the number of rounds remaining and the incumbent's value. So, when the incumbent is the same as in the previous round, the threshold s^* falls (e.g., round 3 in the first game and round 4 in the second game) as the expected profits of an entrant if he beats the incumbent rise because he will face less competition in the future. On the other hand, s^* does not depend on the level of the standing bid given the incumbent's value, because it has no effect on the entrant's profits if he beats the incumbent in a knockout, because the standing bid must be below the incumbent's value. The examples also show what happens to the standing bid in different cases. In round 2 of the first game, the incumbent does not face entry, so there is no change in the standing bid because incumbents do not place additional jump bids. There would also have been no change in the standing bid if the entrant had come in (e.g., if his signal was 100) because the entrant's value was below the current bid, so the standing bid would not have risen in the knockout. In round 3 of the first game, the standing bid rises during the knockout, and the *new* incumbent places an additional jump bid. On the other hand, in round 3 of the second game, the standing bid rises during the knockout phase, but there is no additional jump bid because the old incumbent wins the knockout.

2.3.5   Effect of the Parameters on Equilibrium Strategies

To give some intuition for how equilibrium bid functions and entry probabilities are affected by the parameters, consider a new incumbent whose value is distributed with density proportional to $LN(4.5, 0.2)$ on $[0, 200]$ in the penultimate round of the game, and the standing bid (the value of the previous incumbent) is 80. We consider how the values of σ_ε, K and the location parameter of the value distribution for the final firm affect the bid function and the probability of entry in the *subsequent* final period.
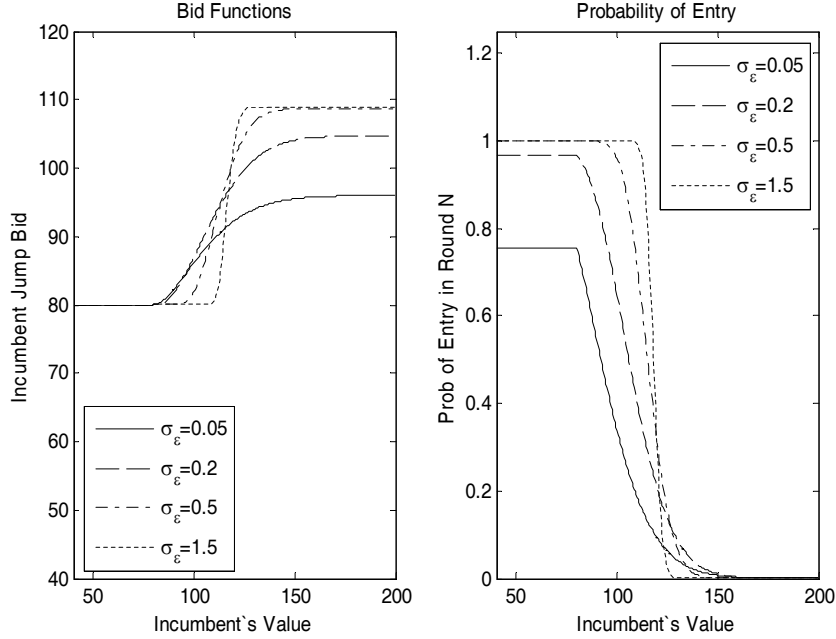
Figure 1: Penultimate Round Equilibrium Bid Functions for a New Incumbent and Associated Final Round Entry Probabilities : Symmetric firms, $K{=}1$, initial standing bid 80

In Figure 1 the precision of the signal (σ_ε) varies. When the signal is very imprecise, both the bid function and the entry probability approximate step functions. If there were no signals (BK's model) then there would be an exact step function in both the bid function and the probability of entry function. When signals are more precise, the slope of the bid function is determined by how much more entry an incumbent could deter by submitting a slightly higher bid - the more entrants who would be deterred, the steeper the bid function must be in equilibrium for firms to truthfully reveal their values when bidding. As a result, when signals are precise the bid function starts rising significantly above the standing bid for relatively low incumbent values (because with precise signals more low value entrants can be discouraged from entering by slightly higher bids), whereas for mean incumbent values (the mean of the distribution is 91, and the mean conditional on having a value more than the standing bid is 99.8), bid functions tend to be flatter (because, with precise signals high value entrants are unlikely to be deterred from entering by slightly higher bids). The probabilities of entry for the firm in the final round are consistent with these arguments: when signals are precise, the probability of entry falls more smoothly with the incumbent's value. Overall there tends to be more entry when signals are imprecise, which reflects the greater option value of entry to a firm that does not know its value (but does know the incumbent's value). On the other hand, the probability of entry by a firm with a high value will tend to be higher when signals are more precise, and these entrants are more valuable to the seller.

In Figure 2 , the level of K (entry cost) varies In this case the comparative statics are simple.
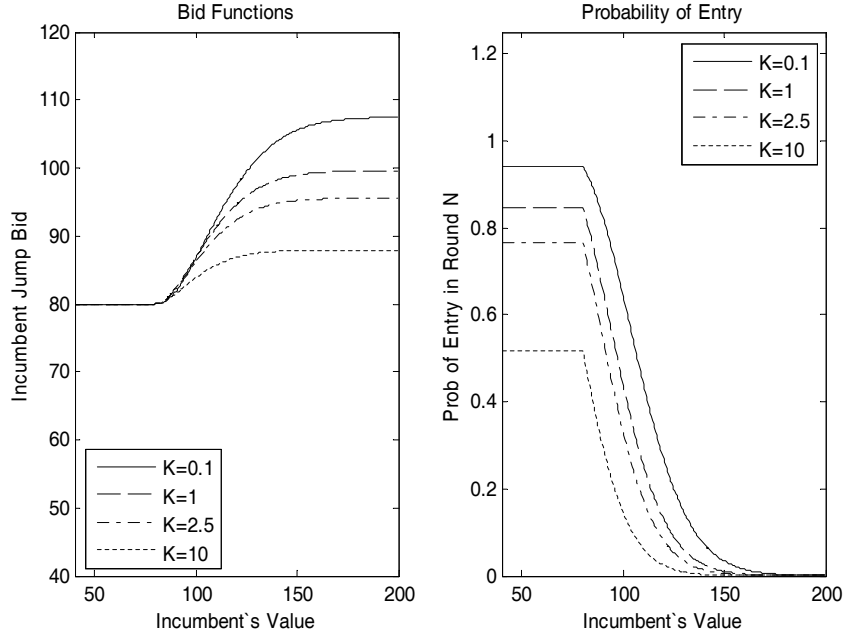
Figure 2: Penultimate Round Equilibrium Bid Functions for a New Incumbent and Associated Final Round Entry Probabilities : Symmetric firms, $\sigma_\varepsilon = 0.1$, initial standing bid 80

When K is higher, there is less of an entry threat, which reduces the incumbent's value of submitting a higher bid to deter entry resulting in a flatter bid function. The probability of entry falls monotonically in K.

Figure 3 shows the effect when the value distribution of the final potential entrant has a lower location parameter ($K = 1, \sigma_\varepsilon = 0.1$). When it has a lower distribution, future entry is less likely because a weaker firm is less likely to beat the incumbent. This makes the incumbent's bid function flatter, at least when the standing bid is 80, which is high relative to the mean value of the potential entrant when $\mu_2 = 4.1$.

3 Comparison of Expected Revenues

Before introducing specific parameters estimated from data for USFS timber auctions, we present a more general comparison of expected revenues and efficiency between the sequential mechanism and the simultaneous entry auction. We see this general comparison as valuable, because it shows that out results are not going to be particularly sensitive to the parameters that we estimate, and they also provide guidance about when auctions should perform well in other settings.

We focus on how the performance of the mechanisms depends on the level of entry costs (K) and the level of the precision of the signal. We measure the precision of the signal by a parameter, $\alpha = \frac{\sigma_\varepsilon^2}{\sigma_\varepsilon^2 + \sigma_V^2}$, which approximately measures the weight a potential entrant puts on its prior when
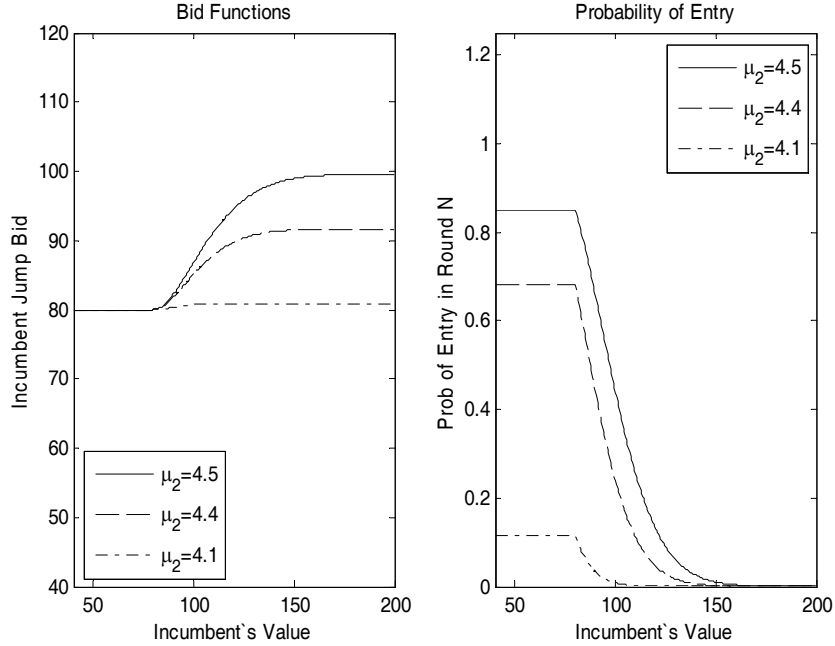
Figure 3: Penultimate Round Equilibrium Bid Functions for a New Incumbent and Associated Final Round Entry Probabilities : Asymmetric firms, $\sigma_\varepsilon = 0.1$, $K = 1$, initial standing bid 80

forming his posterior belief about its log(value).[12] A higher value of α means that signals are less precise: as α approaches 1 the informational assumptions approach those of BK's model. As a base case, we consider 4 symmetric firms whose values are distributed $LN(4.5, 0.2)$. Figure 4 shows the results of comparing expected revenues from the sequential mechanism (with no reserve) and a simultaneous entry auction with no reserve, based on a grid of points in (K, α) space.[13] Filled blue circles represent outcomes where the expected revenues from the sequential mechanism are higher by more than 4% (of auction revenues), while hollow blue circles are outcomes where they are higher but only by between 1 and 4%. Red circles represent cases where the auction gives higher revenues. Black crosses on the grid mark locations where the difference in revenues is less than 1%. As there is some simulation error in calculating expected revenues and some numerical approximations in solving the differential equations in the sequential game, we see these are outcomes as cases where any difference in the revenues is small and we may not be completely confident about their signs (the 1% band is conservative). Note that the grid points are not uniformly distributed in α space: instead, we sampled more points for very low and very high values of α to see how revenues compare when we approximate models with no signals or perfectly informative signals.

The results indicate that for very low values of K, the difference in expected revenues is small.

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[12]Ignoring the upper bound $\overline{V}$, a potential entrant's posterior distribution for its log value will be $N(\alpha\mu + (1 - \alpha)\log(s), \alpha\sigma_V^2)$ where μ is the location parameter for its prior and s is its signal.

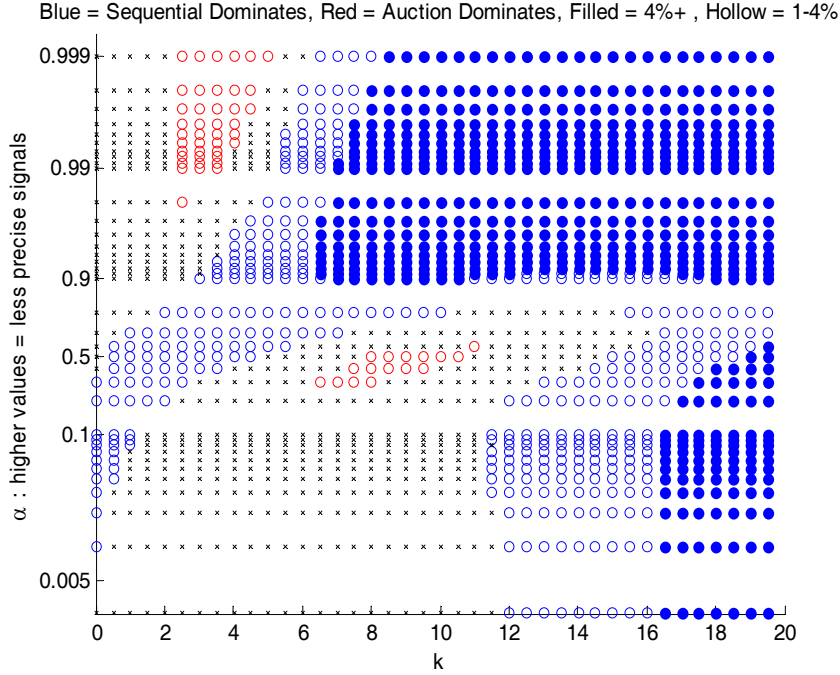[13]Expected revenues are calculated using 200,000 simulations.

Figure 4: Expected Revenue Comparison : 4 Symmetric Firms, values LN(4.5,0.2), No Reserve Prices for either Mechanism.

This should be expected as when entry costs are low it is very likely that the firms with the two highest values enter, so that the final price will be equal to the value of the second highest firm in either mechanism (firms in the sequential mechanism submit relatively low jump bids because deterrence is ineffective when the entry cost is small). Revenues are also similar when signals are quite informative and K is not too large (if it is large, the sequential mechanism does better). There are two small regions of the parameter space where the auction produces higher expected revenues, although the revenue advantage of the auction is never particularly large (the maximum difference found is 3.3%). The area in the top-left corresponds to cases where entry costs are low and signals are uninformative. These points are consistent with BK's theoretical results as their model assumes no signals and their assumptions restrict them to consider cases where it is guaranteed that a certain number of firms will enter the auction, so implicitly K must be quite low. There is also a region where the auction dominates in the middle of the parameter space where signals are moderately informative and entry costs are moderately high. For high levels of K the sequential mechanism clearly dominates, especially for very low or very high values of α.

Figure 5 compares revenues when the seller-optimal reserve is set in the auction but the sequential mechanism has no reserve.[14] As one would expect, there are more parameters where the auctions dominates in this case, but the changes are quite small, reflecting the fact noted above

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[14]The optimal reserve is found using a grid search, with 25 cent increments and simulated revenues.

that reserves are fairly ineffective at raising revenues in auctions with endogenous entry.
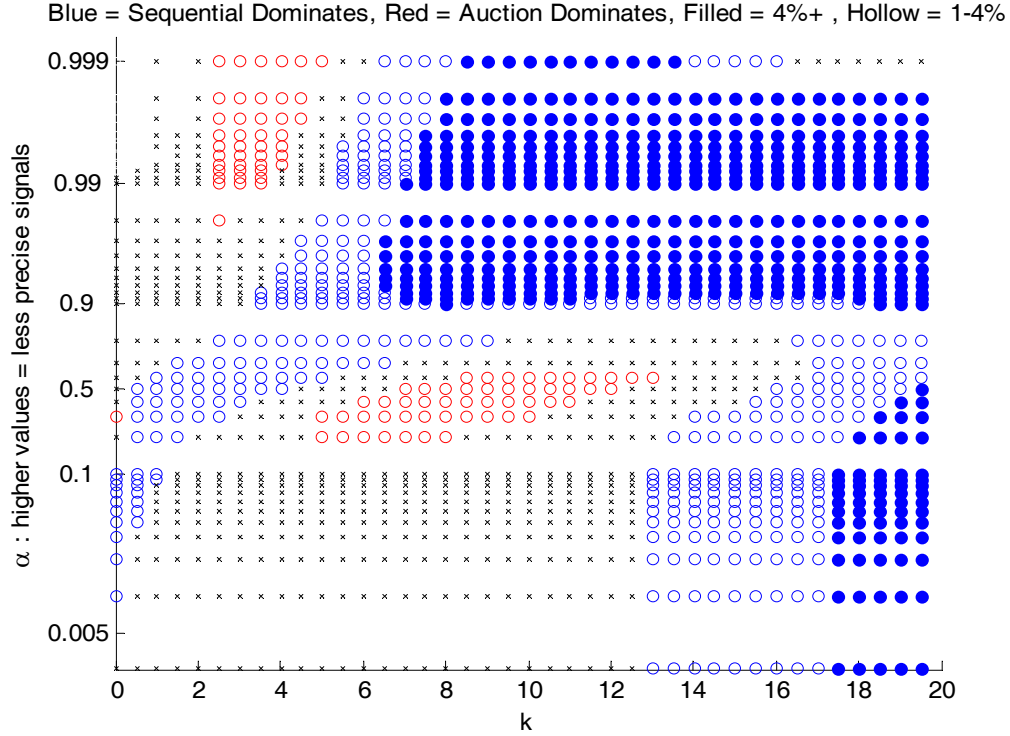

Figure 5: Expected Revenue Comparison : 4 Symmetric Firms, values LN(4.5,0.2), Optimal Reserve Price in Auction, No Reserve in Sequential Mechanism

To explain why the sequential mechanism sometimes outperforms the auction, it is useful to compare the equilibrium efficiency of each mechanism and to understand how aggressively firms have to bid in the sequential mechanism to reveal their values. As BK note, in a model without selective entry and low entry costs, the sequential mechanism is always more (socially) efficient, but the seller may extract lower revenues if there is too much deterrence (from its perspective) and deterring bids are too low.

Figure 6 (a) shows how the efficiency of the auction and the sequential mechanism compare as a function of α (the degree of selection) for $K = 1$ and $K = 5$ for our baseline parameters when there is no reserve (this increases the efficiency of the auction). Efficiency is measured by the expected value of the firm receiving the good less total entry costs. The figure shows two features of the model which appear to be true in general (considering lots of parameter values). First, when entry costs are higher the relative efficiency of the sequential mechanism is greater (recall, BK's assumptions constrain entry costs to be fairly small). This reflects the fact that high entry costs

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make the feature of the sequential mechanism that more firms tend to enter only when existing entrants have low values more socially valuable. In fact, with any selection, the final potential entrant enters only if its expected value is greater than the value of the incumbent less entry costs, which is the efficient criterion for entry. Second, while a small increase in selection causes a small decline in the efficiency of the sequential mechanism when α is very high (little selection), in general more selective entry increases the efficiency advantage of the sequential mechanism. For example, when $K = 5$ the sequential mechanism is 7% more efficient. To raise revenues, the seller only has to be able to appropriate some of this increase in surplus using the sequential mechanism.
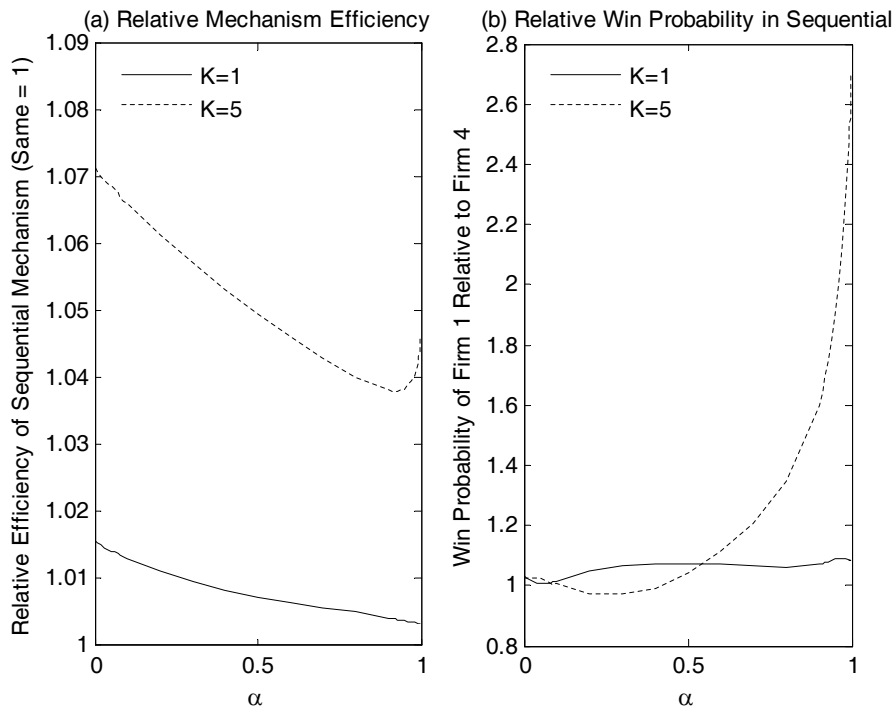
Figure 6: Comparison of Mechanism Efficiency and the Relative Probability that Different Firms in the Sequential Mechanism Win : 4 Symmetric firms values LN(4.5,0.2) on [0,200]

One reason why the sequential mechanism is more efficient with selection is that it is more likely the good is allocated to the firm with the highest value whatever its position in the order chosen by the seller. Figure 6 (b) shows the relative probability that the first and last firms in the sequential mechanism are allocated the good in equilibrium when the order is chosen randomly. If the good was always allocated to the firm with the highest value then these probabilities would be equal to 1, and it will also be equal to 1 in the simultaneous entry auction where the order is irrelevant. On the other hand, entry deterrence in the sequential mechanism tends to move the probability above 1, with early buyers more likely to receive the good. Counterbalancing this tendency is the possibility that the last firm may be more likely to enter in the sequential mechanism because it

faces no future competition, and so does not have to submit a jump bid. When $K = 1$ incumbents only deter entry when they have very high values so that the relative probabilities are close to 1, independent of the degree of selection. On the other hand, $K = 5$, the relative probability is still fairly close to 1 even for values of α such as 0.6 or 0.7 that imply only moderate levels of selection - on the other hand, when α is higher later firms are less likely to win which, all else equal, reduces efficiency.[15]

While the relative efficiency of the sequential mechanism increases with the degree of selection, revenues are also affected by how aggressively firms bid. When there is selection, the level of bids is determined by the fact that bids must be sufficiently high that firms with lower values will not want to copy them. In particular, if the entry decisions of later potential entrants are likely to be sensitive to beliefs about the incumbent's value (which is true when entry costs are moderately high and signals are informative), equilibrium bid functions can be quite steep functions of the incumbent's value, and this can raise revenues. An interesting illustration of this point comes from comparing the equilibrium bid function in the sequential mechanism when there is no selection (BK's assumption) with the equilibrium bid functions in our model when potential entrants receive signals but they are not very informative. In Figure 7, the bid function of the LS model is a step function, which increases at a value of 119 (the level of the incumbent's value that deters all future entry). The bid functions with signals lie above this bid function for all incumbent values, which, holding entry decisions in the last round constant must tend to increase revenues. [16]

Of course, the fact that the sequential mechanism can lead to less (but more efficient) entry than the auction can still hurt the seller, and this helps to explain the second region (intermediate values of α, moderately high K) where the auction gives higher expected revenues. For example, when $K = 8$ and $\alpha = 0.4$, the probability that each firm enters the auction is 0.56, but the probabilities that the firms enter the sequential mechanism are only 0.33, 0.32, 0.30, 0.27 for each firm in the order respetively, and the probability that only one firm enters is 0.76. In this case, the single entrant does not bid aggressively enough to offset the loss of competition through reduced entry. For example, with probability 0.144 the last firm is the only firm to enter the sequential mechanism and it wins at a price equal to zero.

Figure 8 compares revenues when there are 8 rather than 4 firms (with symmetry and no reserve price in the auction). With more firms, the auction only gives higher revenues when entry costs are really low and α is very high (non-selective entry), and, in particular, it never dominates when there is a reasonably selective entry process. This reflects the fact that, even with some selection, simultaneous entry decisions are relative inefficient and this becomes more important when there

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[15]Whether the probability would be equal to 1 in the *optimal* (sequential search) mechanism would depend on how the mechanism was structured. If the seller was able to truthfully elicit signals from the buyers before visiting them, then the relative probabilities for the initial (signal discovery) should equal 1. If on the other hand, the seller could only visit potential buyers one at a time then the relative probability would be above 1 for the optimal mechanism as well. Note also that the relative probability being above 1 does not imply that the first potential buyer has higher expected profits in equilibrium. In fact, the expected profits of the final potential buyer tend to be higher unless α is very high, while the expected profits of all earlier buyers are fairly similar.

[16]As σ_ε is increased to very high values (such as 20 or 30) the equilibrium bid function falls so that it becomes very close to the equilibrium bid function without signals.
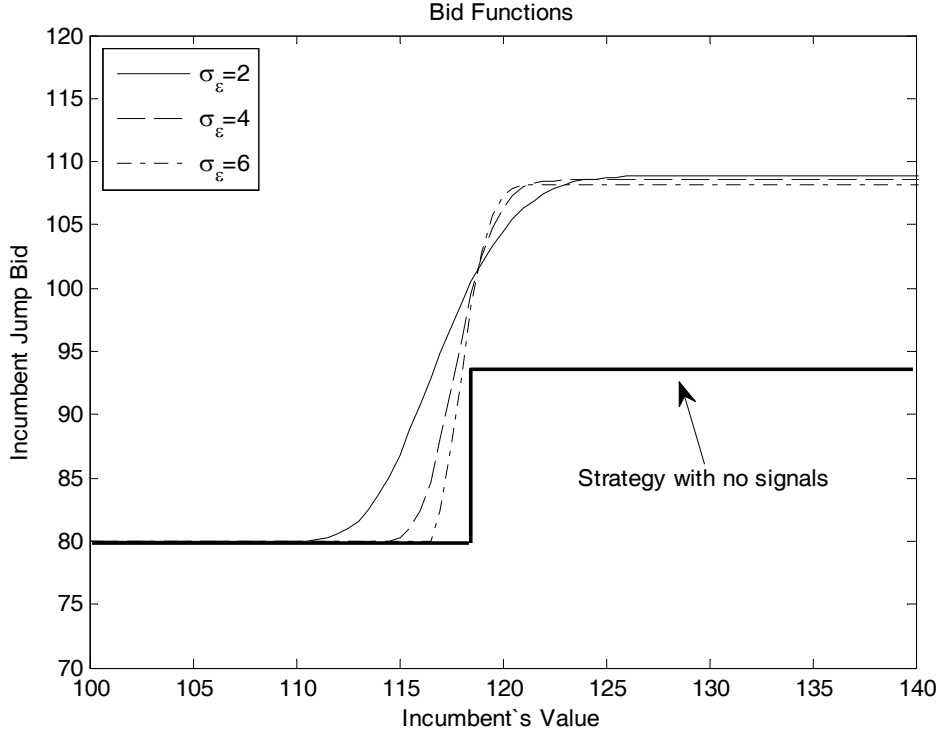
Figure 7: Penultimate Round Bid Function for A New Incumbent : Symmetric firms values LN(4.5,0.2) on [0,200], K=1, Initial standing bid 80

are more firms, so that, at least when entry costs are not very small, the incremental effect of additional firms on revenues is larger in the sequential mechanism. Figure 9 illustrates this point: when $K = 5$ the efficiency advantage of the sequential mechanism is at least a couple of percentage points higher than with 4 firms.

Finally we consider the effect of introducing a small asymmetry in values. Two potential entrants have values drawn from a distribution with density proportional to $LN(4.5, 0.2)$. In the sequential mechanism these firms are approached first.[17] The other two firms have values drawn from a distribution with density proportional to $LN(4.4, 0.2)$. Figure 10 shows the comparison when the auction has no optimal reserve price. [18] Even with only a small asymmetry in values, expected revenues are significantly higher using the sequential mechanism for a much broader range of values than was the case with 4 symmetric firms. In particular, the sequential mechanism always

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[17]Our simulations show that approaching all of the high value firms first, followed by all of the low values firms is better than doing the exact opposite. However, we have not yet checked whether mixing the types in someway is better than either of these alternatives.

[18]In future revisions we will consider the performance of a first-price auction using optimal reserves for different types of bidder. In a model without selective entry, such auctions would do better than simple second-price auctions. Some initial simulations suggest that in our model first-price auctions will do better than second price auctions but not by much.
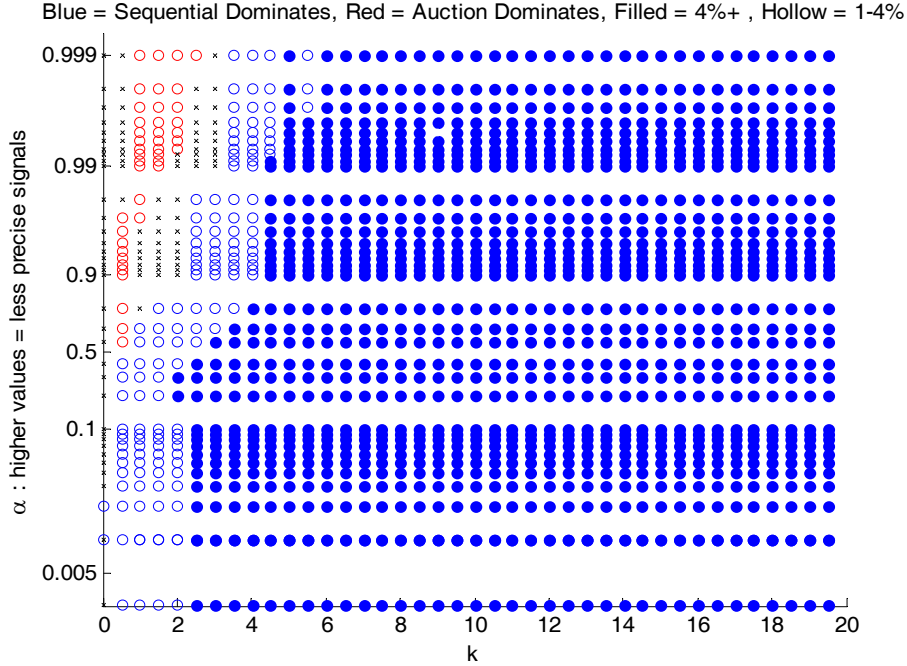
Figure 8: Expected Revenue Comparison : 8 Symmetric Firms, values LN(4.5,0.2), Optimal Reserve Price in Auction, No Reserve in Sequential Mechanism.

does at least 1% better when entry is relatively selective and K is above the minimum value we consider (0.01).

One reason why the sequential mechanism does better when firms are asymmetric and higher mean value firms move first is that the weaker-type firms make their entry decisions knowing the value of any incumbent higher value firms. Relative to the simultaneous entry case, this tends to make them more likely to enter, which raises the probability that they win, and also induces the higher mean value entrants to bid more aggressively. For example suppose that $K = 8$ and $\alpha = 0.4$, a case which favored the auction with symmetric firms. In the simultaneous-entry auction the probability that each of the weaker firms enters is 0.1 and the probability that one of them wins is only 0.12. On the other hand, in the sequential mechanism the entry probabilities are 0.18 and 0.16 (even though they move last) and the probability that one of them wins is 0.30. This is much closer to the probability that one of the weaker firms will have the highest value (0.33 given our distributional assumptions).

3.1 Sequential Mechanism with Reserve Prices

One feature of the sequential mechanism that sellers should value is the relative ease with which it can be implemented. It imposes the same informational demands on the seller as running the auction, except that an exhaustive set of potential buyers must be identified. Moreover, even if
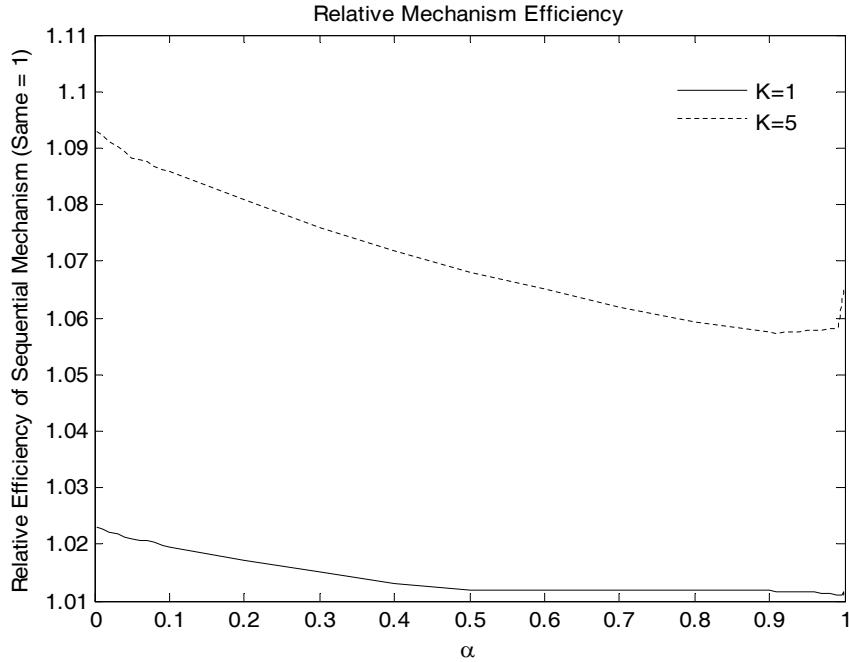
Figure 9: Comparison of Mechanism Efficiency : 8 Symmetric firms values LN(4.5,0.2) on [0,200]

the auction's optimal reserve price is used, which requires the seller to have accurate knowledge of all of the model's parameters, the sequential mechanism with no reserve price will still frequently earn higher revenues for the seller (e.g. Figures 5, 8 or 10). However, adding a reserve price to the sequential mechanism can increase the amount by which it already outperforms the auction. This is shown in Figure 11, which computes the seller's expected revenues when an optimal reserve price is added to each mechanism when there are four or eight symmetric bidders. For the sequential mechanism, only one reserve price is used, which is constant across all rounds in the mechanism. Generally, the seller could do better with a round-specific reserve price, but we view a constant reserve price for the entire mechanism as approximately imposing the same informational demands on the seller as does setting the optimal reserve price in the simultaneous mechanism.

Figure 11 shows that adding a constant reserve price to the sequential mechanism can sometimes substantially improve revenues. This contrasts with the small change in revenues when the optimal reserve price is used for the simultaneous mechanism (see Roberts and Sweeting (2011) for more on this result). The reserve price's impact on sequential mechanism revenues is sensibly smaller when N is greater. It is also less when entry is less selective since the overall amount of entry tends to increase in α.

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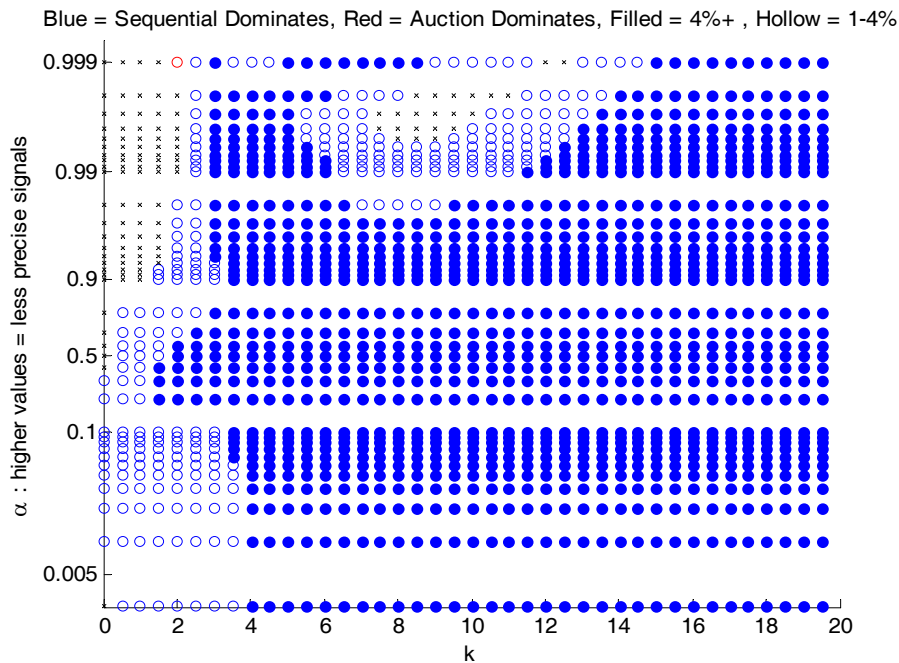
Figure 10: Expected Revenue Comparison : 4 Asymmetric Firms, values LN(4.5 or 4.4,0.2), Optimal Reserve Price in Auction, No Reserve in Sequential Mechanism. Higher Value Firms Visited First in Sequential Mechanism.

4 Empirical Application

We now turn to our empirical application and describe the data, reduced form evidence of selection in these auctions and the method and assumptions we use to estimate a selective entry model using data from open outcry auctions. We are rather brief here since Roberts and Sweeting (2011) provides a more detailed discussion of these topics.

4.1 Data

We analyze federal auctions of timberland in California.[19] In these auctions the USFS sells logging contracts to individual bidders who may or may not have manufacturing capabilities (mills and loggers, respectively). When the sale is announced, the USFS provides its own "cruise" estimate of the volume of timber for each species on the tract as well as estimated costs of removing and processing the timber. It also announces a reserve price and bidders must submit a bid of at least this amount to qualify for the auction. After the sale is announced, each bidder performs its own private cruise of the tract to assess its value. These cruises can be informative about the tract's

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[19]We are very grateful to Susan Athey, Jonathan Levin and Enrique Seira for sharing their data with us.

Expected Revenues in SPA and Sequential Mechanism

Values [0,200], LN(4.5,0.2), K=5, Alpha=($\sigma_\epsilon^2/\sigma_\epsilon^2+\sigma_v^2$)
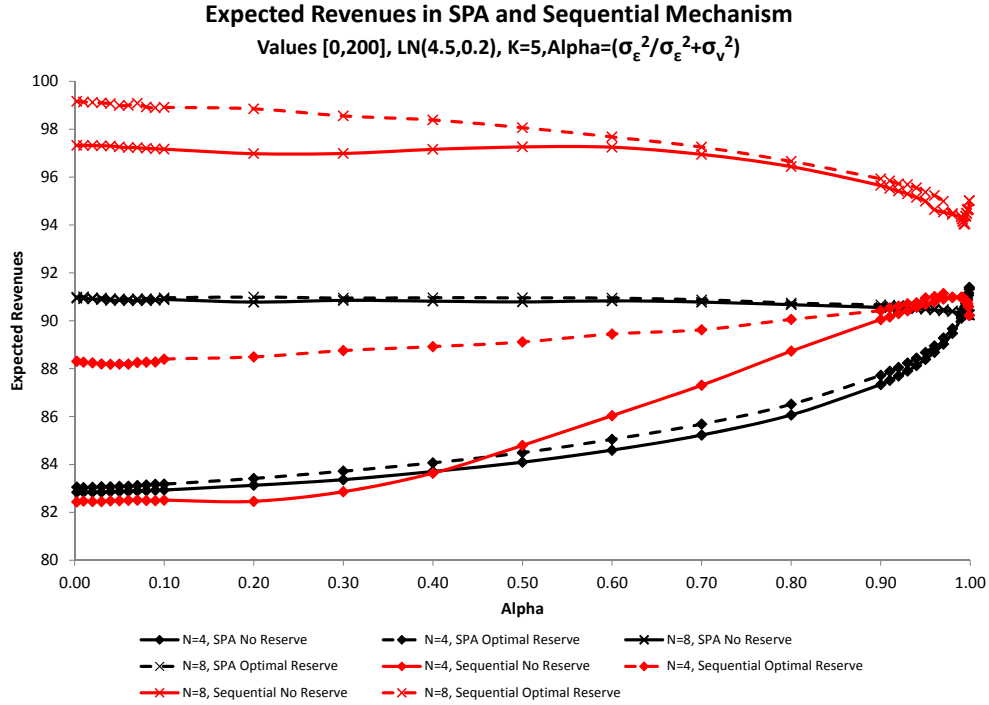
Figure 11: Expected Revenue Comparison: Symmetric Firms, values LN(4.5,0.2) and $K = 5$. The red lines correspond to the sequential mechanism and the black to the auction. The "×"s correspond to the eight bidder case, and the "diamonds" to the four bidder case. The solid lines correspond to mechanisms with no reserve price and the dashed lines to mechanisms with optimal reserve prices.

volume, species make-up and timber quality.

We assume that bidders have independent private values. This assumption is also made in other work with similar timber auction data (see for example Baldwin, Marshall, and Richard (1997), Brannman and Froeb (2000), Haile (2001) or Athey, Levin, and Seira (forthcoming)). A bidder's private information is primarily related to its own contracts to sell the harvest, inventories and private costs of harvesting and thus is mainly associated only with its own valuation. In addition, we focus on the period 1982-1989 when resale, which can introduce a common value element, was limited (see Haile (2001) for an analysis of timber auctions with resale).

We also assume non-collusive bidder behavior. While there has been some evidence of bidder collusion in open outcry timber auctions, Athey, Levin, and Seira (forthcoming) find strong evidence of competitive bidding in these California auctions.

Our model assumes that bidders receive an imperfect signal of their value and there is a participation cost that must be paid to enter the auction. Participation in these auctions is costly for numerous reasons. In addition to the cost of attending the auction, a large fraction of a bidder's entry cost is its private cruise, and people in the industry tell us that firms do not bid without

doing their own cruise. There are several reasons for this. First, some information that bidders find useful, such as trunk diameters, is not provided in USFS appraisals. In addition, the government's reports are seen as useful, but noisy estimates of the tract's timber. For example, using "scaled sales", for which we have data on the amount of timber removed from the tract, we can see that the government's estimates of the distribution of species on any given tract are imperfect. The mean absolute value of the difference between the predicted and actual share of timber for the species that the USFS said was most populous is 3.42% (std. dev. 3.47%).

We use data on ascending auctions. From these, we eliminate small business set aside auctions, salvage sales and auctions with missing USFS estimated costs. To eliminate outliers, we also remove auctions with extremely low or high acreage (outside the range of [100 acres, 10000 acres]), volume (outside the range of [5 hundred mbf, 300 hundred mbf]), USFS estimated sale values (outside the range of [\$184/mbf, \$428/mbf]), maximum bids (outside the range of [\$5/mbf, \$350/mbf]) and those with more than 20 potential bidders (which we define below).[20] We keep auctions that fail to sell. We are left with 887 auctions.

Table 2 shows summary statistics for our sample. Bids are given in \$/mbf (1983 dollars). The average mill bid is 20.3% higher than the average logger bid. As suggested in Athey, Levin, and Seira (forthcoming), mills may be willing to bid more than loggers due to cost differences or the imperfect competition loggers face when selling felled timber to mills.

The median reserve price is \$27.77/mbf. Reserve prices in our sample are set according to the "residual value" method, subject to the constraint that 85% of auctions should end in a sale.[21] In our sample 5% of auctions end in no sale.

We define potential entrants as the auction's bidders plus those bidders who bid within 50 km of an auction over the next month. One way of assessing the appropriateness of this definition is that 98% of the bidders in any auction also bid in another auction within 50 km of this auction over the next month. The median number of potential bidders is eight (mean of 8.93) and this is evenly divided between mills and loggers.

We define entrants as the set of bidders we observe at the auction, even if they did not submit a bid above the reserve price.[22] The median number of mill and logger entrants are three and one, respectively. Among the set of potential logger entrants, on average 21.5% enter, whereas on average 66.1% of potential mill entrants enter.

Our model assumes that differences in values explain why mills are more likely than loggers to enter an auction. However, this pattern could also be explained by differences in entry costs, which we allow to vary across auctions, but not across mills and loggers within an auction. This is unlikely for three reasons. First, there is little reason to believe that the cost of performing cruises

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[20]For estimation we set $\overline{V} = \$500$/mbf, which is substantially above the highest price observed in our sample.

[21]Essentially, the USFS constructed an estimate of the final selling value of the wood and then subtracted logging, transportation, manufacturing and other costs required to generate a marketable product to arrive at a reserve price. The value and cost estimates are based on the government's cruise. See Baldwin, Marshall, and Richard (1997) for a detailed discussion of the method.

[22]This is the definition in Table 2 and we do estimate our model using this definition. However, in our preferred specification, we interpret the data more cautiously and allow bidders that do not submit bids to have entered (paid K), but learned that their value was less than the reserve price.

Variable	Mean	Std. Dev.	25$^{\text{th}}$-tile	50$^{\text{th}}$-tile	75$^{\text{th}}$-tile	N
WINNING BID ($/mbf)	86.01	62.12	38.74	69.36	119.11	847
BID ($/mbf)	74.96	57.68	30.46	58.46	105.01	3426
LOGGER	65.16	52.65	26.49	49.93	90.93	876
MILL	78.36	58.94	32.84	61.67	110.91	2550
LOGGER WINS	0.15	0.36	0	0	0	887
FAIL	0.05	0.21	0	0	0	887
ENTRANTS	3.86	2.35	2	4	5	887
LOGGERS	0.99	1.17	0	1	1	887
MILLS	2.87	1.85	1	3	4	887
POTENTIAL ENTRANTS	8.93	5.13	5	8	13	887
LOGGER	4.60	3.72	2	4	7	887
MILL	4.34	2.57	2	4	6	887
SPECIES HHI	0.54	0.22	0.35	0.50	0.71	887
DENSITY (hundred mbf/acre)	0.21	0.21	0.07	0.15	0.27	887
VOLUME (hundred mbf)	76.26	43.97	43.60	70.01	103.40	887
HOUSING STARTS	1620.80	261.75	1586	1632	1784	887
RESERVE ($/mbf)	37.47	29.51	16.81	27.77	48.98	887
SELL VALUE ($/mbf)	295.52	47.86	260.67	292.87	325.40	887
LOG COSTS ($/mbf)	118.57	29.19	99.57	113.84	133.77	887
MFCT COSTS ($/mbf)	136.88	14.02	127.33	136.14	145.73	887

Table 2: Summary statistics for sample of California ascending auctions from 1982-1989. All monetary figures in 1983 dollars. Here, ENTRANTS are the set of bidders we observe at the auction, even if they did not submit a bid above the reserve price. We count the number of potential entrants as bidders in the auction plus those bidders who bid within 50km of an auction over the next month. SPECIES HHI is the Herfindahl index for wood species concentration on the tract. SELL VALUE, LOG COSTS and MFCT COSTS are USFS estimates of the value of the tract and the logging and manufacturing costs of the tract, respectively. In addition to the USFS data, we add data on (seasonally adjusted, lagged) monthly housing starts, HOUSING STARTS, for each tract's county.

differs substantially across firms for any sale since all potential entrants (a) must attend the auction if they want to submit a bid and (b) are interested in similar information when performing their own cruise (even if their values are different). Second, Table 2 clearly shows that mills also bid more than loggers, suggesting that the meaningful distinction between the types are their value distributions. Third, conditional on entering, loggers are still much less likely to win than mills (15.5% vs. 27.9%). This last point argues against the possibility that loggers enter and win less because of high (relative to mills) entry costs, as this would lead them to enter only when they expect to win.

4.2 Evidence of Selection

In this subsection we argue that the data is best explained by a model that allows for potential selection. First, in the type-symmetric mixed strategy equilibrium of a model with non-selective entry (i.e. the LS model) and asymmetric bidder types, whenever the weaker type enters with positive probability, the stronger type enters with probability one. Thus, for any auction with some

logger entry, a model with no selection would imply that all potential mills entrants enter. In 54.5% of auctions in which loggers participate, and there are some potential mill entrants, some but not all mills participate. A model with selective entry can rationalize partial entry of both bidder types into the same auction.

Second, a model without selection implies that bidders are a random sample of potential entrants. We can test this by applying a Heckman selection model (Heckman (1976)). This consists of a first stage probit of entry as a function of tract characteristics and a flexible polynomial of potential other mill and logger entrants. Using the predicted probabilities, we form an estimate of the Inverse Mills Ratio and include it in the second stage bid regression. The exclusion restriction is that potential competition affects a bidder's decision to enter an auction, but has no direct effect on values. The results appear in Table 3. Column (1) shows OLS results from regressing all bids on auction covariates and whether the bidder is a logger. Column (2) shows the selection model's results. The positive and significant coefficient on the inverse Mills ratio is consistent with bidders being a positively selected sample of potential entrants. In addition, comparing the coefficient on LOGGER across the columns illustrates that selection masks the difference between logger and mill values.[23] This is expected when only those loggers whose values are likely to be high participate and most mills enter ($S'^*_{\text{mill}} < S'^*_{\text{logger}}$).

The evidence presented in this section strongly suggests that the entry process is selective. However, it does not pin down the degree of selection, much less guarantee that the S model is appropriate. Therefore, we now describe how we estimate our model to measure the degree of selection.

4.3 Estimation Using Importance Sampling

To take the model to data, we need to specify how the parameters of the model may vary across auctions, as a function of observed auction characteristics and unobserved heterogeneity. Both types of heterogeneity are likely to be important as the tracts we use differ greatly in observed characteristics, such as sale value, size and wood type, and they also come from different forests so they are likely to differ in other characteristics as well. Both observed and unobserved heterogeneity may affect entry costs and the degree of selection, as well as mean values.

Our estimation approach is based on Ackerberg (2009)'s method of simulated maximum likelihood with importance sampling. Although Roberts and Sweeting (2011) contains a more detailed description of this estimation method and Monte Carlo studies showing that it performs well even with smaller sample sizes than we use, here we briefly summarize the approach.

This method involves solving a large number of games with different parameters once, calculating the likelihoods of the observed data for each of these games, and then re-weighting these likelihoods

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[23]An alternative way to test for selection is to regress bids on covariates, including measures of potential entrants. If there is selection, the potential entrants should have a positive impact on submitted bids as the entry threshold will increase. We have run these regressions, using both all bids and the winning bid, and consistently find a positive impact of potential entrants (in logs or levels). The result also holds if we instrument for potential entrants using lagged participation of mills in nearby auctions.

	(1)	(2)
CONSTANT	-5.475***	-5.792***
	(0.849)	(0.852)
LOGGER	-.090***	-.203***
	(0.026)	(0.04)
SCALE SALE	0.003	-.017
	(0.054)	(0.054)
SPECIES HHI	0.025	0.064
	(0.056)	(0.057)
DENSITY	0.016	0.013
	(0.063)	(0.063)
VOLUME	0.0003	0.0002
	(0.0003)	(0.0003)
HOUSING STARTS	0.0002**	0.0002*
	(0.00008)	(0.00008)
log SALE VALUE	2.750***	2.775***
	(0.081)	(0.081)
log LOG COSTS	-1.052***	-1.093***
	(0.066)	(0.067)
log MFCT COSTS	-.262*	-.181
	(0.147)	(0.148)
$\widehat{\lambda}$		0.159***
		(0.044)
R^2	0.4297	0.4319
N	3,426	3,426

Table 3: Selection evidence. In both columns the dependent variable is log of the bid per volume and year dummies are included. Column (2) displays the second stage results of a two step selection model. The first stage probit is of entry where the exogenous shifters are potential other mill and logger entrants, incorporated as a flexible polynomial. $\widehat{\lambda}$ is the estimated inverse Mills ratio from the first stage.

during the estimation of the distributions for the structural parameters. This method is attractive when it is believed that the parameters of the model are heterogeneous across auctions and it would be computationally prohibitive to re-solve the model (possibly many times in order to integrate out over the heterogeneity) each time one of the parameters changes.[24]

To apply the method, we assume that the parameters are distributed across auctions according to the following distributions, where X_a is a vector of observed auction characteristics and $TRN(\mu, \sigma^2, a, b)$ is a truncated normal distribution with parameters μ and σ^2, and upper and lower

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[24]Hartmann (2006), Hartmann and Nair (forthcoming) and Wang (2010) provide applications of these methods to consumer dynamic discrete choice problems. Bajari, Hong, and Ryan (2010) use a related method to analyze entry into a complete information entry game with no selection.

truncation points a and b.

$$\text{Location Parameter of Logger Value Distribution: } \mu_{a,\text{logger}} \sim N(X_a\beta_1, \omega^2_{\mu,\text{logger}})$$

$$\text{Difference in Mill/Logger Location Parameters: } \mu_{a,\text{mill}} - \mu_{a,\text{logger}} \sim TRN(X_a\beta_3, \omega^2_{\mu,\text{diff}}, 0, \infty)$$

$$\text{Scale Parameter of Mill and Logger Value Distributions: } \sigma_{Va} \sim TRN(X_a\beta_2, \omega^2_{\sigma_V}, 0.01, \infty)$$

$$\alpha: \alpha_a \sim TRN(\beta_4, \omega^2_\alpha, 0, 1)$$

$$\text{Entry Costs: } K_a \sim TRN(X_a\beta_5, \omega^2_K, 0, \infty)$$

These specifications reflect our assumptions that σ_V, α and K are the same for mills and loggers within any particular auction, even though they may differ across auctions. The lower bound on σ_{Va} is set slightly above zero simply to avoid computational problems that were sometimes encountered when there was almost no dispersion of values.[25] Our estimated specifications also assume that the various parameters are distributed independently across auctions. This assumption could be relaxed, although introducing a full covariance matrix would significantly increase the number of parameters to be estimated and, when we have tried to estimate these parameters, we have not found these coefficients to be consistently significant across specifications. The set of parameters to be estimated are $\Gamma = \{\beta_1, \beta_2, \beta_3, \beta_4, \beta_5, \omega^2_{\mu,\text{logger}}, \omega^2_{\mu,\text{diff}}, \omega^2_{\sigma_V}, \omega^2_\alpha, \omega^2_K\}$, and a particular draw of the parameters $\{\mu_{a,\text{logger}}, \mu_{a,\text{mill}}, \sigma_{Va}, \alpha_a, K_a\}$ is denoted θ.

Denoting the outcome for an observed auction by y_a, the log-likelihood function for a sample of A auctions is

$$\sum_{a=1}^{A} \log \left(\int L_a(y_a|\theta)\phi(\theta|X_a, \Gamma)d\theta \right) \tag{4}$$

where $L_a(y_a|\theta)$ is the likelihood of the outcome y in auction a given structural parameters θ, $\phi(\theta|X_a, \Gamma)$ is the pdf of the parameter draw θ given Γ, our distributional assumptions, the unique equilibrium strategies implied by our equilibrium concept and auction characteristics including the number of potential entrants, the reserve price and observed characteristics X_a.

Unfortunately, the integral in (4) is multi-dimensional and cannot be calculated exactly. A natural simulation estimator would be

$$\int L_a(y_a|\theta)\phi(\theta|X_a, \Gamma)d\theta \approx \frac{1}{S} \sum_{s=1}^{S} L_a(y_a|\theta_s) \tag{5}$$

where θ_s is one of S draws from $\phi(\theta|X_a, \Gamma)$. The problem is that this would require us to make new draws of θ_s and re-solve the model S times for each auction in our data each time one of the

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[25]The problem arises from the fact that numerically solving the game requires calculating expected player profits on a grid of possible values, and numerical problems are encountered when the numerical value calculated for value distribution's pdf is non-zero at only one or two grid points. In practice, when taking draws of the parameters for solving the games, we are also imposing upper truncation points on all of the parameters. We choose large values for these, and varying them has no significant effects on our estimates.

parameters in Γ changes. Instead we follow Ackerberg by recognizing that

$$\int L_a(y_a|\theta)\phi(\theta|X_a,\Gamma)d\theta = \int L_a(y_a|\theta)\frac{\phi(\theta|X_a,\Gamma)}{g(\theta|X_a)}g(\theta|X_a)d\theta \tag{6}$$

where $g(\theta|X_a)$ is the importance sampling density whose support does not depend on Γ, which is true in our case because the truncation points are not functions of the parameters. This can be simulated using

$$\frac{1}{S}\sum_s L_a(y_a|\theta_s)\frac{\phi(\theta_s|X_a,\Gamma)}{g(\theta_s|X_a)} \tag{7}$$

where θ_s is a draw from $g(\theta|X_a)$. Critically, this means that we can calculate $L_a(y_a|\theta_s)$ for a given set of S draws that do not vary during estimation, and simply change the weights $\frac{\phi(\theta_s|X_a,\Gamma)}{g(\theta_s|X_a)}$, which only involves calculating a pdf when we change the value of Γ rather than re-solving the game.

This simulation estimator will only be accurate if a large number of θ_s draws are in the range where $\phi(\theta_s|X_a,\Gamma)$ is relatively high, and, as is well known, simulated maximum likelihood estimators are only consistent when the number of simulations grows fast enough relative to the sample size. We therefore proceed in two stages. First, we estimate Γ using $S = 2,500$ where $g(\cdot)$ is a multivariate uniform distribution over a large range of parameters which includes all of the parameter values that are plausible. Second, we use these estimates $\widehat{\Gamma}$ to repeat the estimation using a new importance sampling density $g(\theta|X_a) = \phi(\theta_s|X_a,\widehat{\Gamma})$ with $S = 500$ draws per auction. The Appendix provides Monte Carlo evidence that the estimation procedure works well even for smaller values of S.

To apply the estimator, we also need to define the likelihood function $L_a(y_a|\theta)$ based on the data we observe about the auction's outcome, which includes the number of potential entrants of each type, the winning bidder and the highest bids announced during the open outcry auction by the set of firms that indicated that they were willing to meet the reserve price. A problem that arises when handling data from open outcry auctions is that a bidder's highest announced bid may be below its value, and it is not obvious which mechanism leads to the bids that are announced (Haile and Tamer (2003)).

In our baseline specification we therefore make the following assumptions that we view as conservative interpretations of the information that is in the data: (i) the second highest observed bid (assuming one is observed above the reserve price) is equal to the value of the second-highest bidder[26]; (ii) the winning bidder has a value greater than the second highest bid; (iii) both the winner and the second highest bidder entered and paid K_a; (iv) other firms that indicated that they would meet the reserve price or announced bids entered and paid K_a and had values between the reserve price and the second highest bid; and, (v) all other potential entrants may have entered (paid K_a) and found out that they had values less than the reserve, or they did not enter (did not pay K_a). If a firm wins at the reserve price we assume that the winner's value is above the reserve price.

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[26]Alternative assumptions could be made. For example, we might assume that the second highest bidder has a value equal to the winning bid, or that the second highest bidder's value is some explicit function of his bid and the winning bid. In practice, 96% of second highest bids are within 1% of the high bid, so that any of these alternative assumptions give similar results. We have computed some estimates using the winning bid as the second highest value and the coefficient estimates are indeed similar.

5 Empirical Results

In this section we present estimates of our structural model and counterfactual results measuring the benefits to the USFS from switching their allocation mechanism to a sequential process.

5.1 Parameter Estimates

Table 4 presents the parameter estimates for our structural model.[27] We allow the USFS estimate of sale value and its estimate of logging costs to affect mill and logger values and entry costs since these are consistently the most significant variables in regressions of reserve prices or winning bids on observables, including controls for potential entry, and in the specifications in Table 3. We also control for species concentration since our discussions with industry experts lead us to believe that this matters to firms. We allow for auction-level unobserved heterogeneity (to the econometrician) in all parameters. The rightmost columns show the mean and median values of the structural parameters when we take 10 simulated draws of the parameter for each auction. For the rest of the paper, we refer to these as the "mean" and "median" values of the parameters. All standard errors are based on a non-parametric bootstrap with 100 repetitions.

The coefficients show that tracts with greater sale values and lower costs are more valuable, as one would expect. It does appear that there is both unobserved heterogeneity in values across auctions (the standard deviation of μ_{logger}) and heterogeneity in the difference between mill and logger mean values across auctions (the standard deviation of $\mu_{\text{mill}} - \mu_{\text{logger}}$).

Based on the mean value of the parameters, the mean values of mill and logger potential entrants are \$61.95/mbf and \$42.45/mbf, respectively, a 46% difference. Figure 12 shows the value distributions for potential entrants of both types.
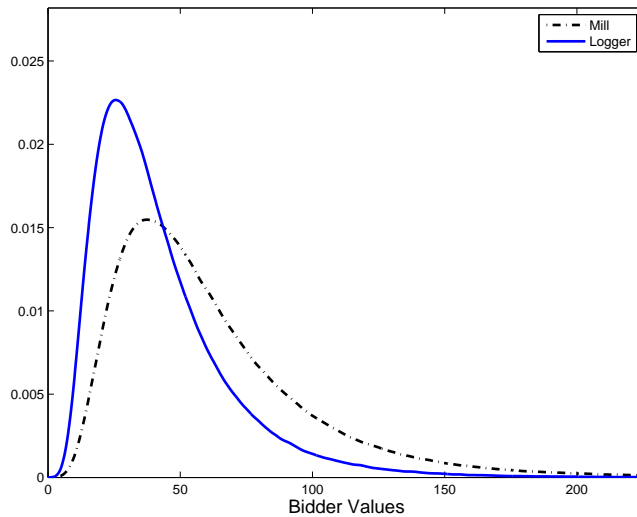

Figure 12: Comparing the value distributions for mills (dash-dot) and loggers (solid). Based on the mean value of the parameters from Table 4.

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[27]Roberts and Sweeting (2011) discuss alternative estimation methods that were attempted, such as Nested Pseudo-Likilhood, and model fit.

Parameter	β parameters				ω parameter	Mean	Median
	Constant	log SELL VALUE	log LOG COSTS	SPECIES HHI			
$\mu_{a,\text{logger}}$ $\sim N(X_a\beta_1, \omega^2_{\mu,\text{logger}})$	-9.6936 (1.3690)	3.3925 (0.1911)	-1.2904 (0.1332)	0.2675 (0.1386)	0.3107 (0.0213)	3.5824 (0.0423)	3.5375 (0.0456)
$\mu_{a,\text{mill}} - \mu_{a,\text{logger}}$ $\sim TRN(X_a\beta_3, \omega^2_{\mu,\text{diff}}, 0, \infty)$	3.6637 (0.8890)	-0.4998 (0.1339)	-0.0745 (0.0919)	-0.1827 (0.1007)	0.1255 (0.0163)	0.3783 (0.0242)	0.3755 (0.0249)
σ_{Va} $\sim TRN(X_a\beta_2, \omega^2_{\sigma_V}, 0.01, \infty)$	4.0546 (0.7872)	-0.7379 (0.0994)	0.1393 (0.1025)	0.0895 (0.0813)	0.0796 (0.0188)	0.5763 (0.0273)	0.5770 (0.0302)
α_a $\sim TRN(\beta_4, \omega^2_\alpha, 0, 1)$	0.7127 (0.0509)	-	-	-	0.1837 (0.0446)	0.6890 (0.0362)	0.6992 (0.0381)
K_a $\sim TRN(X_a\beta_5, \omega^2_K, 0, \infty)$	1.9622 (13.2526)	-3.3006 (2.7167)	3.5172 (2.4808)	-1.1876 (1.5721)	2.8354 (0.6865)	2.0543 (0.2817)	1.6750 (0.3277)

Table 4: Simulated maximum likelihood with importance sampling estimates allowing for non-entrants to have paid the entry cost. The rightmost columns show the mean and median values of the structural parameters when we take 10 simulated draws of the parameter for each auction. Standard errors based non-parametric bootstrap with 100 repetitions. $TRN(\mu, \sigma^2, a, b)$ is a truncated normal distribution with parameters μ and σ^2, and upper and lower truncation points a and b. Based on 887 auctions.

We estimate a mean entry cost of \$2.05/mbf. One forester we spoke with estimated modern day cruising costs of approximately \$6.50/mbf (in 2010 dollars). Converting our estimate to 2010 dollars yields an estimate of \$4.49/mbf. This estimate is consistent with information costs being the majority of entry costs. It is sensible that our estimate is less than the forester's estimate if firms in our data are able to use any information they learn when deciding whether to enter other auctions.

Our estimation approach assumes that, if there are multiple equilibria, the firms will play the equilibrium where mills have the lower S'^*. We checked whether our parameter estimates can support multiple equilibria by plotting type-symmetric "equilibrium best response functions" for mills and loggers for each auction. For each auction, our parameter estimates support only a single equilibrium. This is because our estimates imply a large difference in the mean values of loggers and mills, relatively low entry costs and a moderate amount of selection (which we discuss in more detail below).

The assumptions used to generate our preferred results in Table 4, which we use in all counter-factuals below, are based on a conservative interpretation of what we know about the entry decisions of firms that do not attend the auction. Roberts and Sweeting (2011) also show estimates if we instead assume that firms that did not attend the auction did not pay K. The main differences are slightly (i) more selection and (ii) higher entry costs. This is sensible since we are now assuming that the only firms that enter are those we see and they have values greater than the reserve. A lower α and higher K are needed to rationalize this.

Our estimates of the αs across auctions indicate a moderate amount of selection in the data. Based on our estimates, we find a 46% difference in mean values for potential mill and logger entrants. This is much larger than the average difference in bids across mills and loggers (20.3%), as would be expected if entry is selective. If we consider a representative auction where the reserve and the number of potential mill and logger entrants are set to their respective medians of \$27.77/mbf, four and four, we can compare the difference in values between marginal (those who observed $S'^*_{\tau a}$) and inframarginal bidders. Based on the mean parameter values, the average mill entrant's value is \$68.13/mbf and the average marginal mill bidder's value is \$45.22/mbf. The fact that the average potential mill entrant's value is higher than the average marginal mill's value reflects the fact that most mills enter. The comparable numbers for entrant and marginal loggers are \$59.80/mbf and \$48.13/mbf, respectively. The difference between marginal and inframarginal bidders is indicative of the degree of selection in the entry process. Also note that, for these estimates, marginal loggers tend to have higher values than marginal mills. For illustration, Figure 13 compares the entrants' and marginals' value distributions for each bidder type in the representative auction based on the mean parameters. Due to selection, there is a substantial difference in the marginal and inframarginal bidders for each type.
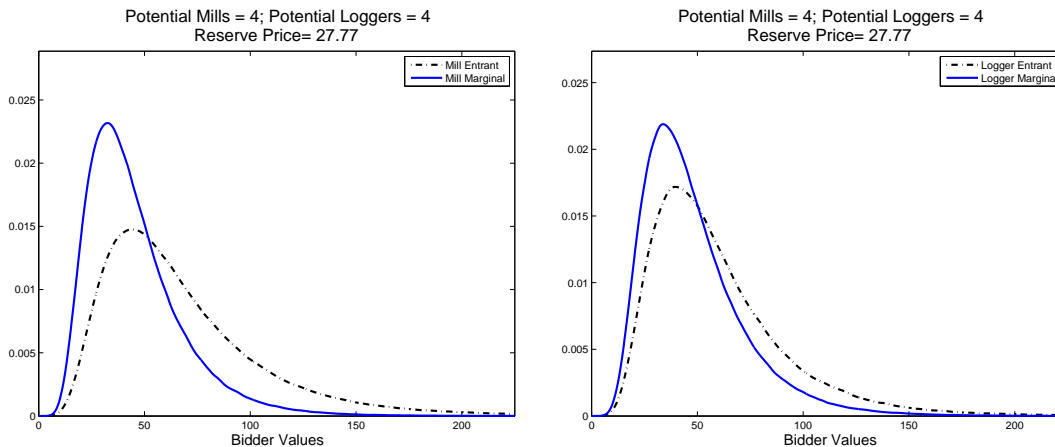
Figure 13: Comparing the value distributions for entrant and "marginal" bidders by type. Based on the mean value of the parameters from Table 4.

5.2 Counterfactual Results

Table 5 compares expected revenues from the sequential mechanism and the simultaneous entry auction for a range of parameters and different numbers of firms. We compute expected revenues in the auction for an auction with no reserve, and for an auction with an optimal reserve. These are always close, and when we use observed reserves the results lie in between these estimates.

The first line in Table 5 gives the results for the representative auction used in the constructing the figures above. Relative to setting no reserve, the sequential mechanism improves the USFS's revenues by 4.8%. This is much larger than the improvement from using an optimal reserve in this auction, which is just 0.2%. The sequential mechanism is projected to give 4.48% higher revenues than a second price auction with an optimal reserve price. For a tract of average size (7,626 mbf) the expected revenue difference would be $24,022.

The other lines in the table compare revenues when we increase or decrease the parameters by one standard deviation (the changing parameters are in *italics*), reflecting the fact that our estimates imply that the coefficients will differ across sales. The cases we consider indicate that using the sequential mechanism always raises the seller's expected revenues, with especially large increases when there are more loggers (weaker firms), higher entry costs and more precise signals (more selective entry), which are the broad patterns that we would expect from the simulations in Section 3. In all cases, the revenue increases from using the sequential mechanism appear to be much larger than the returns to using a (single) reserve price in a second-price auction, even though understanding optimal reserve price policies for timber auctions has been the subject of significant interest in the literature (Paarsch (1997), Haile and Tamer (2003), Aradillas-Lopez, Gandhi, and Quint (2010a)).

We also consider the USFS switching to a first price auction with type-specific reserves. We

view this only as a useful theoretical benchmark given that we are unaware a first price auction with type specific reserve prices actually being implemented in practice. Even compared to this auction format with optimal reserve prices, the sequential mechanism, with no reserve price at all, still performs well. Moreover, a reserve price can appreciably increase revenues in the sequential mechanism, as evidenced in Figure 11. For example, in the representative auction, the optimal reserve price for the sequential mechanism is approximately \$49.00/mbf. With this reserve price, expected seller revenues increase to \$76.58. This represents a 7.2% and 2.9% increase in revenues over a second price auction with an optimal reserve price and a first price auction with type specific optimal reserves, resepecitvely.

| | | Parameters | | | | | | Expected Revenues (\$/mbf) | | | | Sequential Adv. over |
| | | | | | | | | | SPA | | FPA | SPA: R^* |
N_{mill}	N_{logger}	μ_{logger}	μ_{diff}	σ_V	α	K	Sequential	$R=0$	R^*	$R^*_{\text{Mill}}, R^*_{\text{Logger}}$	
4	4	3.582	0.378	0.567	0.689	2.05	74.77	71.33	71.47	74.45	4.6%
1	4	3.582	0.378	0.567	0.689	2.05	54.49	50.99	51.67	56.39	5.5%
7	4	3.582	0.378	0.567	0.689	2.05	84.88	84.01	84.08	85.71	1.0%
4	*0*	3.582	0.378	0.567	0.689	2.05	67.33	64.47	64.75	64.38	4.0%
4	*8*	3.582	0.378	0.567	0.689	2.05	79.16	75.95	76.05	78.65	4.1%
4	4	*2.921*	0.378	0.567	0.689	2.05	36.60	34.12	34.37	35.09	6.5%
4	4	*4.243*	0.378	0.567	0.689	2.05	147.38	143.61	143.67	148.89	2.6%
4	4	3.582	*0.169*	0.567	0.689	2.05	64.53	61.96	62.08	62.95	4.0%
4	4	3.582	*0.587*	0.567	0.689	2.05	88.31	84.00	84.17	89.11	4.9%
4	4	3.582	0.378	*0.349*	0.689	2.05	59.75	57.16	57.50	58.47	3.9%
4	4	3.582	0.378	*0.804*	0.689	2.05	95.30	91.13	91.21	-	4.5%
4	4	3.582	0.378	0.567	*0.505*	2.05	75.14	70.71	71.10	73.31	5.7%
4	4	3.582	0.378	0.567	*0.872*	2.05	74.67	71.97	72.00	-	3.7%
4	4	3.582	0.378	0.567	0.689	*0.39*	78.05	75.83	75.84	-	2.9%
4	4	3.582	0.378	0.567	0.689	*3.72*	71.41	66.77	67.20	68.87	6.3%

Table 5: Comparison of Expected Revenues from Simultaneous Entry Auctions and the Sequential Mechanism for Various Hypothetical Timber Sales. The first line shows the results for the representative auction discussed above. Italics indicate changes from this representative auction.

6   Conclusion

This paper compares the performance - from a seller's perspective - of a standard simultaneous entry second price auction with an alternative sequential mechanism in an environment where firms

with higher values may be more likely to enter. We see this model as a plausible description of many environments where auctions are currently used, and potential bidders have to perform costly research to find out their values prior to bidding. We find that the sequential mechanism can do significantly better, which is contrary to what one might expect in light of Bulow and Klemperer (2009)'s results that were based on an entry model which allowed no selection. The revenue advantage of the sequential mechanism is typically larger when entry is more selective, especially when entry costs are low, when entry costs are high and when there are asymmetries between bidders, and so far we have not found any examples where the auction gives significantly higher (e.g., 4% or more) expected revenues. We use our model to examine whether the US Forest Service would do better using the sequential mechanism than the type of auction it currently uses, and find that the gains in revenues are potentially quite large in many representative auctions, and this pattern is robust to choosing different values for the main parameters. The revenue gains from using a different mechanism are generally much larger than those from setting the optimal reserve within the existing auction format. Our current results may well understate the gains to changing the mechanism, as we have only considered one relatively simple alternative mechanism with no reserve. Introducing a reserve price within the sequential mechanism may be more effective than within the simultaneous entry auction as it would raise the starting point for jump bids and, when we simulate the current model, there is often only one entrant who currently wins at a low deterring bid, or, in the case of the final firm, with a bid of zero.

Of course it may be the case that in practice standard auctions have advantages, such as transparency, which our model ignores. It is certainly true that our mechanism would require, for example, a process for establishing the set of potential buyers (maybe by setting a very small fee for indicating a potential interest in bidding) and then a commitment by the seller to approach firms in some random order (at least within-type), so that particular firms are not favored (typically the final firm approached has the highest expected profits because it does not have to jump bid to deter future entry). However, it is not obvious to us that the administrative burden involved would always exceed the quite large gains in revenues which appear to be available from considering different mechanisms, and it seems likely that requiring all firms to send representatives to a room on a particular date to hold a simultaneous auction also imposes some cost on bidders. The informational requirements of the sequential mechanism on the seller are also relatively small: in contrast, the benefits of more complicated auction schemes - involving the provision of subsidies to particular types of bidder, for example - might be dependent on the seller knowing the value distribution, entry costs and signal noise of different types of firm.

Our research highlights one obvious direction for future theoretical research. We take Bulow and Klemperer's sequential mechanism as an off-the-shelf, and reasonably straightforward, alternative to current practice. The fact that a sequential mechanism, and particularly one that involves firms' values being sequentially revealed, performs well is consistent with papers in the optimal mechanism design literature (Cremer, Spiegel, and Zheng (2009) and McAfee and McMillan (1988)) that consider settings where participation by potential buyers is costly, and they either have perfect

or no information about their values prior to entering, and find that mechanisms which implement a sequential search policy are optimal. However, there are currently no models which consider optimal mechanism design when buyers have partially informative signals before they participate, and it is these models which seem most likely to be relevant in practice.

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A Conditions for Unique Sequential Equilibrium Under the D1 Refinement

We now verify the three conditions required to show that our equilibrium is the unique separating equilibrium. For clarity, we repeat the three conditions here.

1. $\frac{\pi_v(b_1, S_2'^*)}{\partial S_2'^*} > 0$;

2. $\frac{\pi_v(b_1, S_2'^*)}{\partial b_1} \Big/ \frac{\pi_v(b_1, S_2'^*)}{\partial S_2'^*}$ is monotonic in v;

3. $S_2'^*$ is uniquely defined for any belief about the first potential entrant's value, and $S_2'^*$ is increasing.

(1) $\frac{\pi_v(b_1, S_2'^*)}{\partial S_2'^*} > 0$: Increasing the signal threshold keeps out more second round potential entrants. The bidding behavior of those entrants who have signals above the threshold is unchanged and so this increases the incumbent's probability of winning and lowers the expected price paid.

(2) $\frac{\pi_v(b_1, S_2'^*)}{\partial b_1} \Big/ \frac{\pi_v(b_1, S_2'^*)}{\partial S_2'^*}$ is monotonic in v: Differentiating gives:

$$\frac{\partial^2 \pi_v(b_1, S_2'^*)}{\partial b_1 \partial v} \left(\frac{\partial \pi_v(b_1, S_2'^*)}{\partial S_2'^*} \right)^{-1} - \left(\frac{\partial^2 \pi_v(b_1, S_2'^*)}{\partial S_2'^* \partial v} \right) \left(\frac{\partial \pi_v(b_1, S_2'^*)}{\partial S_2'^*} \right)^{-2} \frac{\partial \pi_v(b_1, S_2'^*)}{\partial b_1} \tag{8}$$

We need to show that Equation 8 is either always positive or always negative. We do this by establishing (a)-(d) below.

(a) $\frac{\partial^2 \pi_v(b_1, S_2'^*)}{\partial b_1 \partial v} = 0$: Consider two types of first round bidders v_H and v_L, $v_H > v_L$, with each considering increasing their bid b to $b + \varepsilon$. If the second bidder stays out then the cost to each first round type is the same, ε. We now show that if the second round bidder comes in with a value v_2, the cost is still the same to each type of first round bidder. Consider three cases. (i) $v_2 < b$. The cost to each type of first round bidder is ε since each still wins but pays more. (ii) $v_2 > b + \varepsilon$. The cost to each type will be same and equal to zero since the final price in this case is $\min\{v_L, v_2\}$ for the low type and $\min\{v_H, v_2\}$ for the high type. Both are independent of the deterring bid.[28] (iii) $b \leq v_2 \leq b + \varepsilon$. The first round bidder still wins, regardless of type, but now he has to pay more since before he would have won at a price of v_2 but now he wins at a price of $b + \varepsilon$, yielding the same cost of $b + \varepsilon - v_2$ to each type of first round bidder. Therefore, the cost of raising the deterring bid, all else constant, is independent of the first bidder's value.

(b) $\left(\frac{\partial^2 \pi_v(b_1, S_2'^*)}{\partial S_2'^* \partial v} \right) > 0$: To show that the benefit of increasing the signal entry threshold is greater the higher is the first bidder's value, we can show that the benefit of excluding any second bidder type v_2 is greater, the higher is the first bidder type, regardless of v_2. Consider the value of excluding a second round bidder whose value is v_2 for any two types of first round bidders v_H and

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[28]There are three cases within this case. The first is when $v_2 > v_H > v_L$. Regardless of deterring bid, both first round types would lose and so increasing the deterring bid has no effect on their cost. The second is when $v_H > v_2 > v_L$. Here the low type was going to lose regardless, and so it has no effect on his cost. Here the high type was going to win but pay v_2 no matter what and so increasing the bid has no effect on his cost. The third is when $v_H > v_L > v_2$. In either case both types were going to win but have to pay v_2 and so increasing the bid had no effect on either types' costs.

v_L, $v_H > v_L$, both using deterring bid b. If $v_2 \le b$ there is no change in benefit from exclusion for either first bidder type. If $b < v_2$ there are three cases. (i) $v_2 \le v_L < v_H$. In this case the benefit of excluding the second round bidder is $v_2 - b$ for each first round bidder type. (ii) $v_L < v_2 \le v_H$. In this case the benefit of exclusion is $v_L - b$ for the low type and $v_2 - b$ for the high type. Since by assumption $v_2 > v_L$, the benefit of exclusion is greater for the higher type. (iii) $v_L < v_H < v_2$. In this case the benefit of exclusion is $v_L - b$ for the low type and $v_H - b$ for the high type and so the benefit is greater for the higher first bidder type. Therefore, the benefit of excluding more second round bidders is greater the higher is the first round bidder's value.

(c) $\frac{\pi_v(b_1, S_2'^*)}{\partial S_2'^*} > 0$: This was shown above when we verified the first of the three conditions.

(d) $\frac{\partial \pi_v(b_1, S_2'^*)}{\partial b_1} < 0$: Increasing the bid is costly when it does not affect the second round potential entrant's decision. In particular, it reduces a firm's payoff when the second round firm does not enter or it enters and has a value less than b. If the potential entrant enters with a value above b then changing b has no effect.

Combining (a)-(d), we conclude:

$$\underbrace{\frac{\partial^2 \pi_v(b_1, S_2'^*)}{\partial b_1 \partial v} \left(\frac{\partial \pi_v(b_1, S_2'^*)}{\partial S_2'^*} \right)^{-1}}_{=0} - \underbrace{\left(\frac{\partial^2 \pi_v(b_1, S_2'^*)}{\partial S_2'^* \partial v} \right) \left(\frac{\partial \pi_v(b_1, S_2'^*)}{\partial S_2'^*} \right)^{-2} \frac{\partial \pi_v(b_1, S_2'^*)}{\partial b_1}}_{<0} > 0 \qquad (9)$$

and the condition is satisfied.

(3) $S_2'^*$ is uniquely defined for any belief about the first potential entrant's value, and the second potential entrant chooses a better action for the incumbent when he believes that the incumbent's value is higher: this is true in our case, as $S_2'^*$, is a continuous function of the second period potential entrant's beliefs about the incumbent's value (reflecting the zero profit condition and the potential entrant's beliefs about his own value as a function of the signal) and the second potential entrant will increase $S_2'^*$ if he believes bidder 1's type is higher because his expected profits are decreasing in bidder 1's type for any signal he receives.

Competition versus Auction Design

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Abstract

We analyze the value of auction design tools relative to fundamental economic forces by estimating the impact of setting an optimal reserve price and increasing the number of potential bidders in USFS timber auctions. We allow for costly participation that selects entrants on dimensions that affect ex post competition, since both reserve prices and potential competition can change the number and types of bidders that participate. The USFS's value of an optimal reserve price is low, both in absolute terms and relative to even marginal increases in the number of potential bidders.

JEL CODES: D44, L20, L73

Keywords: Auctions, Entry, Selection

1   Introduction

A large theoretical literature provides many insights about how sellers should optimally design auctions by, for example, setting an optimal reserve price. However, there have been few attempts to evaluate the value of these design tools relative to more fundamental economic factors, such as the number of potential bidders (competition). This is especially true in realistic settings where there may be several types of bidders, auction participation is costly and entry is likely to be selective, by which we mean that potential bidders with higher values are more likely to participate, so that changes in reserve prices or increased potential competition may endogenously affect the number and types of participants. This lack of formal empirical evidence is all the more glaring since at least some respected experts, who have advised many sellers on how to design auctions, believe that fundamental factors such as competition may be much more important than many design tools (Klemperer (2002)). In this paper, we compare the relative value to a seller of using reserve prices and increasing competition in a particular empirical setting, United States Forest Service (USFS) timber auctions, where optimal reserve price policies have been the subject of considerable previous study.[1] To do this, we develop and estimate a flexible empirical entry model for independent private value (IPV) second price and ascending auctions that allows us to evaluate the degree of selection in the entry process, and which could be used to evaluate the effects of other policies or changes in market structure that would be likely to affect the entry margin.[2]

We build a model that allows for endogenous and selective entry and bidder asymmetries because these are common features of many settings in which auctions are used and, in this type of an environment, there are no unambiguous theoretical results regarding the relative value of reserve prices and increased competition to sellers. In a classic paper, Bulow and Klemperer (1996) show that a revenue-maximizing seller will always benefit more from having one more bidder than being able to set an optimal reserve price if bidders are symmetric and all bidders must participate. Even with symmetric bidders, this result is not robust to endogenous participation where assumptions about what bidders know about their values when they decide to enter can be crucial. A common assumption is that potential bidders have no information about their values when they decide to pay the entry cost (we refer to this as the LS model after Levin and Smith (1994)), so that the entry process cannot be selective. In this case, expected revenues decline in the number of potential bidders so that a seller dislikes additional potential competition, and trivially prefers the ability to set an

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[1]Some examples of previous studies of timber auction reserve prices include Mead, Schniepp, and Watson (1981), Mead, Schniepp, and Watson (1984), Paarsch (1997), Haile and Tamer (2003), Li and Perrigne (2003) and Aradillas-Lopez, Gandhi, and Quint (2010). All of these papers assume that entry is not endogenous.

[2]The focus on a second price model makes the exposition easier. Our empirical work allows for the fact that the auctions we analyze are actually open outcry auctions.

optimal reserve price (which is equal to zero for a revenue maximizing seller). An alternative extreme assumption is that potential bidders know their values prior to the entry decision (we refer to this as the S model after Samuelson (1985)). In this case, adding a potential bidder may increase or decrease expected revenues (Samuelson (1985) and Menezes and Monteiro (2000)), and it may be more valuable than the ability to set an optimal reserve price (we provide an example below). This lack of a general result also applies to models, like the one that we estimate, where potential bidders only have imperfect signals about their values when they decide to participate. When potential bidders are asymmetric there are no general results concerning the relative value of competition and reserve prices even for models with complete participation or no selection.

In many real-world settings, asymmetric potential bidders are likely to have some, but imperfect, information about their values when they decide to enter the auction, and a significant part of the entry cost will be the research cost that a firm pays to identify its value. In our empirical setting (USFS timber auctions), two types of firms are potential bidders: sawmills, who have their own manufacturing facilities, and logging companies, who do not, and their observed bidding and entry behavior suggests that mills tend to have significantly higher values. A bidder's value for a particular tract will depend on the types of wood on the tract, tract characteristics such as tree diameters and density, and the bidder's own capabilities and contracts for selling either cut timber or processed wood products downstream. A firm's knowledge of the forest, its own contracts and capabilities and the information released by the USFS when announcing the sale, which can be inaccurate and lack information the bidder would like to know, should allow potential bidders to form some rough estimates of their values prior to deciding whether to participate. However, before submitting bids, bidders conduct their own surveys of each tract ("timber cruises") to determine their values precisely, and the cost of the cruise is seen as one of the most important factors that deter firms from participating in timber auctions. This environment is naturally described by a model which lies somewhere between the informational extremes of the LS and S models. This is also likely to be true in other commonly studied auction environments such as highway paving contract procurement auctions, where bidders must calculate how much the project will cost to complete, and OCS oil tract auctions, where bidders conduct expensive seismic surveys to estimate the amount of oil present.

To capture this key feature of the environment, we formulate a two-stage entry model for second price or ascending IPV auctions with asymmetric bidder types. In the first stage, potential bidders simultaneously decide whether to participate in the auction, which entails paying a sunk entry cost that enables them to learn their value exactly. In the second stage, entrants submit bids. We allow for selection by assuming that each potential entrant

gets a noisy, private information signal about its value before taking an entry decision. We therefore refer to our model as the Signal model. In equilibrium, each potential bidder enters if its signal is above a type-specific threshold, and the precision of signals (which is a parameter that we estimate) determines how selective the entry process is. If signals are very uninformative, then the value distributions of entrants, marginal entrants and non-entrants will be similar (little selection) and outcomes will be similar to those of an LS model with asymmetric firms. On the other hand, when signals are very informative, these distributions will be quite different (a great deal of selection) and outcomes will approach those of the S model.

Our estimates indicate that the entry process for timber auctions is moderately selective. For a representative auction, the average mill (logger) entrant's value is 50.7% (24.3%) greater than that of the marginal mill (logger) entrant. Our estimates also imply that the value of setting an optimal reserve is typically small, both in absolute terms and relative to increasing the number of mills (the stronger type) by one. For example, for a representative auction that has four mill and four logger potential entrants, the USFS's expected value of holding the auction increases by only 0.92% when it sets an optimal reserve price rather than a non-strategic reserve price equal to its value of keeping the tract in the event of no sale. In contrast, there is a 6.66% improvement if one mill potential entrant is added, even if the USFS uses a non-strategic reserve: an increase which is 7.21 times as large. This happens even though the addition of another potential entrant makes existing firms less likely to enter because, with selection, the entrants that are lost tend to be less valuable to the seller. The gain from adding one additional logger potential entrant (1.80%) also exceeds the gain from using an optimal reserve price. Moreover, this assumes there are only incremental increases in the number of potential bidders and the result could be stronger if additional potential entrants are added. For example, in the representative auction, we find that if two potential mill entrants are added, the USFS's value of the auction rises 12.51% even if it continues to set a non-strategic reserve price.

Our paper makes two important contributions. Our first contribution is that we provide, to our knowledge, the first empirical analysis of the relative value of a commonly used auction design tool (reserve prices) and a more fundamental economic force (competition) on auction revenues. To the extent sellers are able to choose their auction design, including the reserve price, and affect potential competition (for example, through advertising or timing sales appropriately), understanding the relative relative returns is of central interest to sellers, including governments, corporations and private individuals. Here our empirical estimates are consistent with the view of Klemperer (2002) who suggests that a large amount of auction

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theory focuses on factors that do not greatly impact a seller's revenues in practical settings.[3] Often, increasing potential competition may also be more practical for sellers, as determining the level of the optimal reserve price requires the seller to know for all types of bidder, the number of potential bidders, the distribution of bidder values, the distribution of signals, and entry costs. As noted by Wilson (1987) (leading to what has come to be known as the "Wilson Doctrine"), auction design tools, including optimal reserve price policies, are not practically useful if they assume too much about the information available to the seller.

Our second contribution is that we provide a structural framework, suitable for settings with asymmetric firms and unobserved market (auction) heterogeneity, that can be used to (a) estimate the degree of selection in the entry process and (b) understand how selective entry may affect outcomes in a wide range of counterfactuals in addition to those considered in this paper. We believe that there are many policy-relevant counterfactuals, in both auction and non-auction settings, that are substantively affected by selective entry. As one example, the U.S. Department of Justice and the Federal Trade Commission's *Horizontal Merger Guidelines*, and similar rules in other countries, indicate that a merger may be allowed if sufficient entry would occur within a one to two-year window to force prices back to their pre-merger levels if the merging parties were to raise prices significantly. The probability of future entry, and its ability to constrain price increases, is likely to depend on whether the entry process has already selected the most competitive firms into the market.[4] As a second example, sellers sometimes use bid subsidies, or restrict sales to particular types of buyers, in order to encourage particular types of firm, such as small or minority-owned businesses, to participate.[5] The cost and benefit of these programs will depend on whether the new firms that are drawn in by these programs tend to have lower values than the firms that would have participated without them, which is obviously determined by the degree of selection. Selection's effects on cost-benefit analysis are suggested by the computational examples in Hubbard and Paarsch (2009) that show that, under perfect selection, bid subsidy programs have only small effects on auction participation. As a final and less obvious example, Roberts and Sweeting (2010) use the model presented here to compare the seller's expected revenue from using a simultaneous bid auction (first price or second price), to using a sequential procedure where buyers can place bids in turn, which mimics how valuable items

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[3]Klemperer provides several examples that illustrate his view. For instance, when U.K. commercial television franchises were auctioned in the early 1990s, prices in regions where numerous firms participated sold for between 9 and 16 pounds per capita, whereas in the Midlands region only one firm participated and it paid only one-twentieth of one penny per capita.

[4]Brannman and Froeb (2000) simulate the effect of mergers in timber auctions assuming exogenous participation.

[5]Athey, Coey, and Levin (2011) study set-asides in timber auctions and Krasnokutskaya and Seim (forthcoming) study subsidies in government procurement under the assumption that the entry process is not selective.

such as companies and real estate are often sold (Bulow and Klemperer (2009)). They show that, with selective entry, the sequential procedure can generate significantly higher revenues. Some of the largest gains occur when entry is moderately selective, which our estimates suggest is an empirically relevant case, as opposed to extreme cases of the LS or S models that are the most commonly used models in the literature.

We regard this framework as an important contribution to the empirical entry literature since, while empirical studies of market entry are now common, the existing literature pays little attention to whether the entry process is selective. The key feature of the selective entry process in our model is that the firms that choose to enter are likely to be stronger competitors, which hurts rivals and benefits consumers (or sellers in auctions), than those firms that do not enter. In the non-auction literature, standard entry models (e.g. Berry (1992)), allow for there to be a shock to a firm's payoff from entering a market, which may cause some firms to enter while others do not. However, they assume that this shock does not affect the profits of other firms, so it must be interpreted as affecting sunk costs or fixed costs. Similarly, dynamic entry models (e.g. Ericson and Pakes (1995)) assume that potential entrants are symmetric apart from i.i.d. shocks to their entry costs. Instead, the natural analogue of our model in non-auction settings would be one where firms receive noisy signals about their post-entry marginal costs or qualities.

In the empirical auction literature (see Hendricks and Porter (2007) for a recent survey), most analyses that allow for endogenous entry also assume no selection, consistent with the LS model (e.g. Athey, Levin, and Seira (forthcoming), who examine timber auctions, Bajari and Hortaçsu (2003), Palfrey and Pevnitskaya (2008), Krasnokutskaya and Seim (forthcoming), Li and Zhang (2010), Bajari, Hong, and Ryan (2010) and Ertaç, Hortaçsu, and Roberts (2011)). However, the use of the S model in theoretical work, has led to some empirical interest in whether the entry process is selective.[6] For example, Li and Zheng (2009) compare estimates from both the LS and S models using data on highway lawn mowing contracts from Texas to understand how potential competition may affect procurement costs, and Li and Zheng (2010) test the LS and S models using timber auctions in Michigan. Marmer, Shneyerov, and Xu (2010) extend this literature by testing whether the Li and Zheng (2009) data is best explained by the LS, S or a more general affiliated signal model. They find support for the S and signal models, and they also estimate a simple version of their signal model.

An important limitation of the Li and Zheng (2009), Li and Zheng (2010) and Marmer,

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[6]Matthews (1995), Menezes and Monteiro (2000) and Tan and Yilankaya (2006) analyze theoretical models that assume potential bidders know their values when deciding whether to enter. Ye (2007) presents a model of "indicative bidding" in which bidders can update their assessment of their value upon paying an entry cost. Examples of theoretical papers that allow for endogenous entry, but assume that it is not selective, include French and McCormick (1984), McAfee and McMillan (1987), Tan (1992) and Engelbrecht-Wiggans (1993).

Shneyerov, and Xu (2010) papers is that they assume that potential bidders are symmetric and, in the last one, that there is no unobserved auction heterogeneity (for example, in the good being sold). These are significant limitations as bidder asymmetries are common and often important for interesting counterfactuals, and unobserved auction heterogeneity has been shown to be important in many settings, including timber auctions (Athey, Levin, and Seira (forthcoming) and Aradillas-Lopez, Gandhi, and Quint (2010) for timber and Krasnokutskaya (forthcoming), Roberts (2009), Hu, McAdams, and Shum (2009) in other settings), and may affect the level of optimal reserve prices (Roberts (2009) and Aradillas-Lopez, Gandhi, and Quint (2010)). The estimation method we use in this paper, which builds on the importance sampling procedure proposed by Ackerberg (2009), allows for asymmetric bidders (mills and loggers in our application) and for rich observed and unobserved heterogeneity in all of the structural parameters across auctions. We focus on second price and ascending auctions, although, at the cost of additional computation, the method can be extended to first price or low bid procurement auctions.[7]

The paper proceeds as follows. Section 2 presents our model. Section 3 introduces the empirical context and data. Section 4 describes our estimation method and discusses identification. Section 5 presents our structural estimates. Section 6 uses them to compare the relative benefits to the USFS of increasing potential competition and setting an optimal reserve price and Section 7 concludes. The Appendix contains Monte Carlo studies of our estimation method.

2   Model

We now present our model of entry into auctions and show that equilibria are characterized by a cutoff strategy whereby a bidder only enters an auction when its signal is sufficiently high. The model description assumes that the auction format is second price sealed bid. However, as we assume that bidders have independent private values, equilibrium strategies would be the same in an English button auction. In Section 4 we will explain how we apply our model to data from an open outcry auction.

Consider an auction a with $N_{\tau a}$ potential bidders (firms) of type τ with $\overline{\tau}$ types in total. In our setting $\overline{\tau}$ is 2 and the types are sawmills and loggers. Type τ firm values V are i.i.d. draws from a distribution $F_{\tau a}^{V}(V)$ (with associated pdf $f_{\tau a}^{V}(V)$), which is continuous on an interval $[0, \overline{V}]$. The distribution can depend on the characteristics of the auction, although the support is fixed. Both the $N_{\tau a}$s and the $f_{\tau a}^{V}(V)$s are common knowledge to all potential

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[7]An incomplete working paper by Einav and Esponda (2008) proposes a method for estimating a partially selective entry model for low bid auctions with bidder asymmetries but no unobserved auction heterogeneity.

bidders. In practice, we will assume that the $f_{\tau a}^V(V)$s will be proportional to the pdfs of lognormal distributions with location parameters $\mu_{\tau a}$ and squared scale parameters σ_{Va}^2 on the $[0, \overline{V}]$ interval, and, as a labeling convention, that $\mu_{1a} > \mu_{2a}$. We will also choose a value for $\overline{V}$ which is significantly above the highest price observed in our data.

Firms play a two stage game. In the first (entry) stage, each firm independently decides whether to enter the auction, which requires paying an entry cost K_a. Prior to taking this decision, each firm i receives an independent, private information signal s_i about its value, where $s_i = v_i z_i$, $z_i = e^{\varepsilon_i}$, $\varepsilon_i \sim N(0, \sigma_{\varepsilon a}^2)$. We assume that part of the entry cost is the cost of researching the object being sold, so that a player who pays the entry cost finds out its true value v_i. These entrants can participate in the second (auction) stage of the game, and we assume that only firms that pay the entry cost can do so. Note that the parameters σ_{Va}^2, $\sigma_{\varepsilon a}^2$ and K_a are assumed to be common across the types. We make this assumption for reasons connected with equilibrium selection, which we explain below.

In the second (bidding) stage, entrants submit bids in a second price auction, so that if a bid is submitted above the auction's reserve price, R_a, the object is sold to the bidder with the highest bid at a price equal to the maximum of the second highest bid and the reserve price.

2.1 Equilibrium

Following the literature (e.g. Athey, Levin, and Seira (forthcoming)), we assume that players use strategies that form type-symmetric Bayesian Nash equilibria, where "type-symmetric" means that every player of the same type will use the same strategy. In the second stage, entrants know their values so it is a dominant strategy for each entrant to bid its value. In the first stage, players take entry decisions based on what they believe about their value given their signal. By Bayes Rule, the (posterior) conditional density $g_{\tau a}(v|s_i)$ that a player of type τ's value is v when its signal is s_i is

$$g_{\tau a}(v|s_i) = \frac{f_{\tau a}^V(v) \times \frac{1}{\sigma_{\varepsilon a}} \phi\left(\frac{\ln\left(\frac{s_i}{v}\right)}{\sigma_{\varepsilon a}}\right)}{\int_0^{\overline{V}} f_{\tau a}^V(x) \times \frac{1}{\sigma_{\varepsilon a}} \phi\left(\frac{\ln\left(\frac{s_i}{x}\right)}{\sigma_{\varepsilon a}}\right) dx} \tag{1}$$

where $\phi(\cdot)$ denotes the standard normal pdf.

The weights that a player places on its prior and its signal when updating its beliefs about its true value depend on the relative variances of the distribution of values and ε (signal noise), and this will also control the degree of selection. A natural measure of the relative variances is $\frac{\sigma_{\varepsilon a}^2}{\sigma_{Va}^2 + \sigma_{\varepsilon a}^2}$, which we will denote α_a. If the value distribution were not

truncated above, a player i's (posterior) conditional value distribution would be lognormal with location parameter $\alpha_a \mu_{\tau a} + (1 - \alpha_a)s_i$ and squared scale parameter $\alpha_a \sigma^2_{V\tau a}$.

The optimal entry strategy in a type-symmetric equilibrium is a pure-strategy threshold rule whereby the firm enters if and only if its signal is above a cutoff, $S'^*_{\tau a}$.[8] $S'^*_{\tau a}$ is implicitly defined by the zero-profit condition that the expected profit from entering the auction of a firm with the threshold signal will be equal to the entry cost:

$$\int_{R_a}^{\overline{V}} \left[\int_{R_a}^{v} (v - x)h_{\tau a}(x|S'^*_{\tau a}, S'^*_{-\tau a})dx \right] g_{\tau a}(v|s)dv - K_a = 0 \tag{2}$$

where $g_{\tau a}(v|s)$ is defined above, and $h_{\tau a}(x|S'^*_{\tau a}, S'^*_{-\tau a})$ is the pdf of the highest value of other entering firms (or the reserve price R_a if no value is higher than the reserve) in the auction.

A pure strategy type-symmetric Bayesian Nash equilibrium exists because optimal entry thresholds for each type are continuous and decreasing in the threshold of the other type.

2.2 Marginal and Inframarginal Bidders with Selection

We now illustrate how the model produces selective entry and how the degree of selection depends on α. We do this by examining the difference between the value distributions of inframarginal entrants and those bidders who received a signal just equal to the signal threshold, what we term to be "marginal entrants". This is an important distinction when analyzing any policy that impacts entry margins since it will affect the participation decisions of marginal entrants.

These distributions must be the same in the LS model. In the Signal model, however, the bidder receiving a signal $S = S'^*$ will tend to have lower values than entrants. This is clearly shown in Figure 1 for the symmetric bidder case. The left (right) panel of Figure 1 displays the average value distributions of entrants and marginal bidders in the Signal model with a precise (imprecise) signal and in the LS model. As the signal noise increases, the Signal model approaches the LS model. Likewise, as $\sigma_\varepsilon \to 0$, the Signal model approaches the S model where entrants' value distributions are perfectly truncated at S'^*.

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[8]A firm's expected profit from entering is increasing in its value, and because values and signals are independent across bidders and a firm's beliefs about its value is increasing in its signal, a firm's expected profit from entering is increasing in its signal. Therefore, if a firm expects the profit from entering to be greater (less) than the entry cost for some signal S, it will also do so for any signal $\widetilde{S}$ where $\widetilde{S} > S$ ($\widetilde{S} < S$). As it will be optimal to enter when the firm expects the profit from entering to be greater than the entry costs, the equilibrium entry strategy must involve a threshold rule for the signal, with entry if $S > S'^*_{\tau a}$.
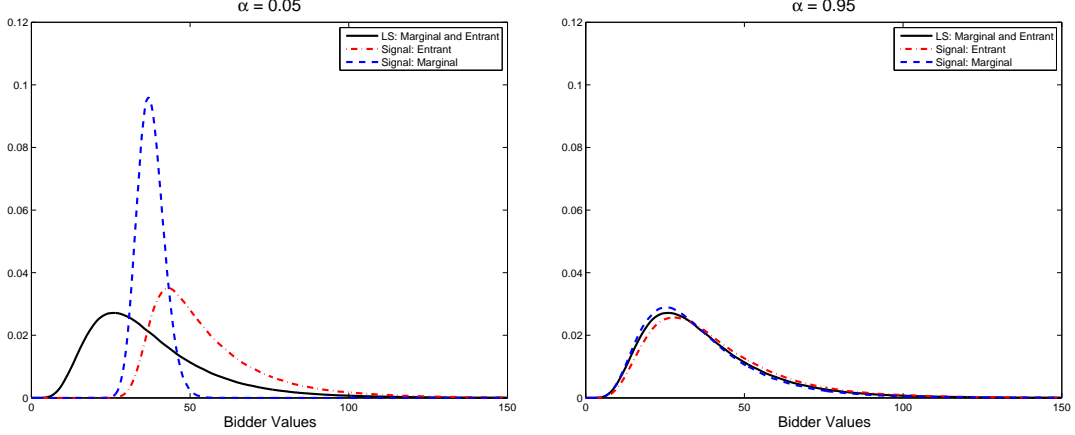
Figure 1: Comparing the value distributions for marginal bidders and entrants in the LS and Signal models. In both panels $V \sim logN(3.5, 0.5)$, $K = 5$, $N = 5$ and $R = 0$. In the Signal model $S = VZ$, $Z = e^{\varepsilon}$, $\varepsilon \sim N(0, \sigma_{\varepsilon}^2)$. The left panel pertains to a case with a high degree of selection: $\sigma_{\varepsilon} = 0.115$ ($\alpha = 0.05$). The right panel makes the comparison when there is less selection: $\sigma_{\varepsilon} = 2.179$ ($\alpha = 0.95$). In both panels the solid lines are for the LS model (in which the marginal and inframarginal bidder value distributions are the same), the dashed line is for the marginal bidder in the Signal model and the dash dot line is for a typical entrant in the Signal model.

2.3 Multiple Equilibria and Equilibrium Selection

Even when we restrict attention to type-symmetric equilibria, a game with more than one type may have multiple equilibria where different types of firm have different thresholds. For example, in our empirical setting, some parameters would support both equilibria where the mills have a lower entry threshold ($S'^*_{\text{mill}} < S'^*_{\text{logger}}$), and equilibria where loggers have a lower threshold ($S'^*_{\text{mill}} > S'^*_{\text{logger}}$).

This is illustrated in the first panel of Figure 2, which shows the reaction functions for the entry thresholds of both types of firm, when there are two firms of each type, $\sigma_V = 0.05$, $K = 4$, $\alpha = 0.1$ ($\sigma_{\varepsilon} = 0.0167$) and $\mu_1 = \mu_2 = 5$, so that the types are actually identical.[9] The reserve price R is set to 20. There are three equilibria (intersections of the reaction functions), one of which has the types using identical entry thresholds (45° line is dotted), and the others involving one of the types having the lower threshold (and so being more likely to enter). The fact that there are at most three equilibria follows from the inverse-S shapes of the reaction functions.

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[9]In this diagram the reaction function represents what would be the symmetric equilibrium best response between the two firms of a particular type when both firms of the other type use a particular S'.

Equilibrium Best Response Functions
$\mu_1 = 5, \mu_2 = 5, \sigma_v = 0.05,$
$\alpha = 0.1 , K = 4, R = 20, N_1 = 2, N_2 = 2$

$\mu_1 = 5.025, \mu_2 = 5, \sigma_v = 0.05,$
$\alpha = 0.1 , K = 4, R = 20, N_1 = 2, N_2 = 2$

$\mu_1 = 5.075, \mu_2 = 5, \sigma_v = 0.05,$
$\alpha = 0.1 , K = 4, R = 20, N_1 = 2, N_2 = 2$
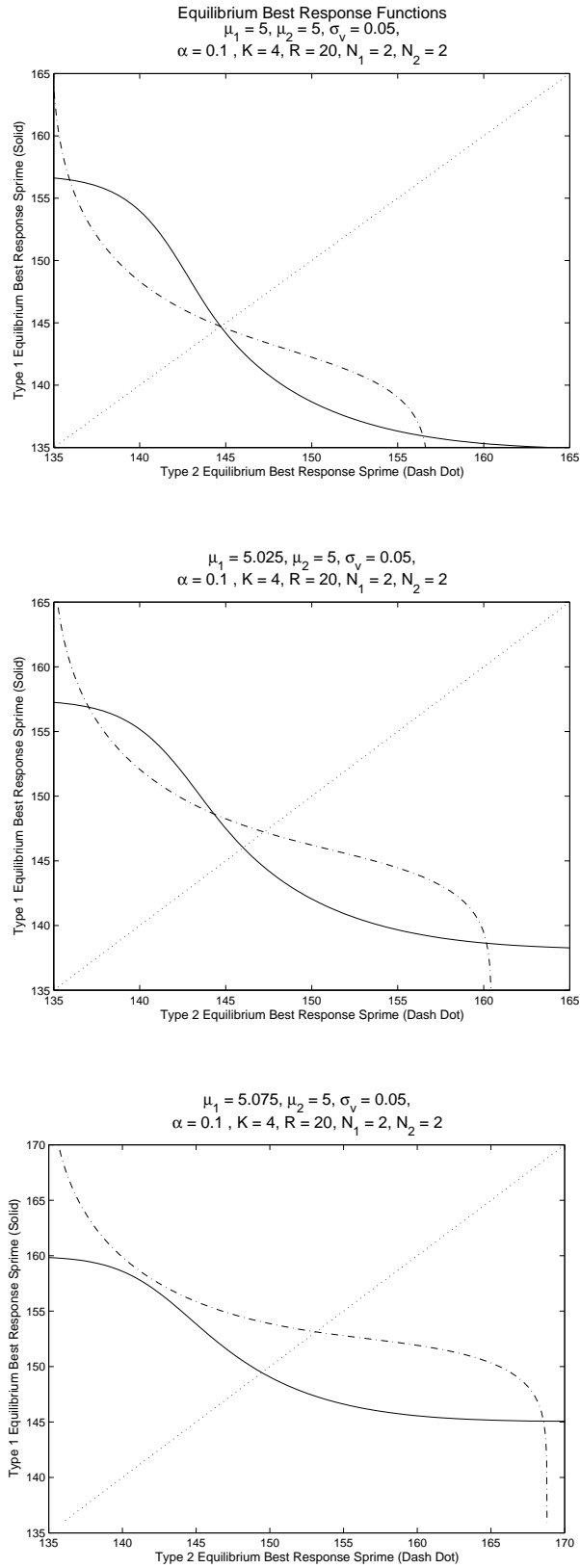
Figure 2: Reaction functions for symmetric and asymmetric bidders. In the first (top) panel, the types are identical so that $\mu_1 = \mu_2 = 5$ and there are two firms of each type, $\sigma_V = 0.05, K = 4, \alpha = 0.1$ ($\sigma_\varepsilon = 0.0167$). In the next two panels we introduce asymmetries in means only and the solid (dash dot) lines correspond to the type with the higher (lower) mean. In the second (middle) panel $\mu_1 = 5.025$ and $\mu_2 = 5$ and the remaining parameters are held fixed. In the third (bottom) panel $\mu_1 = 5.075$ and $\mu_2 = 5$ and the remaining parameters are held fixed. The 45° line is dotted.

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The second panel in Figure 2 shows the reaction functions when we set $\mu_1 = 5.025$ and $\mu_2 = 5$, holding the remaining parameters fixed. This change causes the reaction function of type 1 firms to shift down (for a given S'_2 they wish to enter for a lower signal) and the reaction function of the type 2 firms to shift outwards (for a given S'_1, type 2 firms are less willing to enter). There are still three equilibria, but because of these changes in the reaction functions, there is only one equilibrium where the stronger type 1 firms have the lower entry threshold so that they are certainly more likely to enter. When the difference between μ_1 and μ_2 is increased, there is only one equilibrium and it has this form, as illustrated in the third panel of Figure 2.

The result that there is a unique equilibrium with $S'^*_1 < S'^*_2$ when $\mu_1 \geq \mu_2$ and σ_V, σ_ε and K are the same across types holds generally if the reaction functions have only one inflection point.[10] Under these assumptions it is also generally true that the game has a unique equilibrium, in which it will be the case that $S'^*_1 < S'^*_2$, if $\mu_1 - \mu_2$ is large enough.

The empirical literature on estimating discrete choice games provides several approaches for estimating games with multiple equilibria including assuming that a particular equilibrium is played, estimating a statistical equilibrium selection rule that allows for different equilibria to be played in the data (Sweeting (2009) and Bajari, Hong, and Ryan (2010)) and partial identification tecnhiques that may only give bounds on the parameters (e.g. Ciliberto and Tamer (2009) and Beresteanu, Molchanov, and Molinari (2009)). In this paper we assume that the parameters σ_V, σ_ε and K are the same across types and that, if there are multiple equilibria, the equilibrium played will be the unique one where $S'^*_1 < S'^*_2$. We view our focus on this type of equilibrium as very reasonable, given that it is clear in our data that sawmills (our type 1) tend to have significantly higher average values than loggers (our type 2), so that it is almost certain that only one equilibrium will exist (a presumption that we verify based on our parameter estimates).

In Section 5, we discuss our experimentation with other estimation approaches, such as nested pseudo-likelihood, that make weaker assumptions about the equilibrium that is played or which allow for the types to have different σ_V, σ_ε and K parameters. These approaches give qualitatively similar findings for the degree of selection and for the comparison between the value of reserve prices and potential competition. On the other hand, the advantage of the particular estimation approach that we use (which for practical implementation requires us to make this selection assumption) is that it allows us to control, in a very rich way, for observed and unobserved heterogeneity across auctions, which are clear features of our data.

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[10]In general, the exact shape of the reaction functions depends on the distributional assumptions made for the distributions of values and signal noise. Under our distributional assumptions, we have verified that the reaction functions have no more than one inflection point based on more than 40,000 auctions involving different draws of the parameters and different numbers of firms of each type.

Solving for equilibria in the model is straightforward: we find the S'^* values that satisfy the zero profit conditions for each type and which satisfy the constraint that $S_1'^* < S_2'^*$ using a non-linear equation solver.[11]

3 Data

We analyze federal auctions of timberland in California.[12] In these auctions the USFS sells logging contracts to individual bidders who may or may not have manufacturing capabilities (mills and loggers, respectively). When the sale is announced, the USFS provides its own "cruise" estimate of the volume of timber for each species on the tract as well as estimated costs of removing and processing the timber. It also announces a reserve price and bidders must submit a bid of at least this amount to qualify for the auction. After the sale is announced, each bidder performs its own private cruise of the tract to assess its value. These cruises can be informative about the tract's volume, species make-up and timber quality.

We assume that bidders have independent private values. This assumption is also made in other work with similar timber auction data (see for example Baldwin, Marshall, and Richard (1997), Brannman and Froeb (2000), Haile (2001) or Athey, Levin, and Seira (forthcoming)). A bidder's private information is primarily related to its own contracts to sell the harvest, inventories and private costs of harvesting and thus is mainly associated only with its own valuation. In addition, we focus on the period 1982-1989 when resale, which can introduce a common value element, was limited (see Haile (2001) for an analysis of timber auctions with resale).

We also assume non-collusive bidder behavior. While there has been some evidence of bidder collusion in open outcry timber auctions, Athey, Levin, and Seira (forthcoming) find strong evidence of competitive bidding in these California auctions.

Our model assumes that bidders receive an imperfect signal of their value and there is a participation cost that must be paid to enter the auction. Participation in these auctions is costly for numerous reasons. In addition to the cost of attending the auction, a large fraction of a bidder's entry cost is its private cruise, and people in the industry tell us that firms do not bid without doing their own cruise. There are several reasons for this. First, some information that bidders find useful, such as trunk diameters, is not provided in USFS appraisals. In addition, the government's reports are seen as useful, but noisy estimates of the tract's timber. For example, using "scaled sales", for which we have data on the

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[11]For some of the counterfactuals it is important to do this to a high degree of accuracy, so we set a tolerance of 1e-13. In a first price auction it is necessary to also solve for the equilibrium bid functions given a value of S'^*, which creates additional computational costs.

[12]We are very grateful to Susan Athey, Jonathan Levin and Enrique Seira for sharing their data with us.

amount of timber removed from the tract, we can see that the government's estimates of the distribution of species on any given tract are imperfect. The mean absolute value of the difference between the predicted and actual share of timber for the species that the USFS said was most populous is 3.42% (std. dev. 3.47%).

We use data on ascending auctions. From these, we eliminate small business set aside auctions, salvage sales and auctions with missing USFS estimated costs. To eliminate outliers, we also remove auctions with extremely low or high acreage (outside the range of [100 acres, 10000 acres]), volume (outside the range of [5 hundred mbf, 300 hundred mbf]), USFS estimated sale values (outside the range of [$184/mbf, $428/mbf]), maximum bids (outside the range of [$5/mbf, $350/mbf]) and those with more than 20 potential bidders (which we define below).[13] We keep auctions that fail to sell. We are left with 887 auctions.

Table 1 shows summary statistics for our sample. Bids are given in $/mbf (1983 dollars). The average mill bid is 20.3% higher than the average logger bid. As suggested in Athey, Levin, and Seira (forthcoming), mills may be willing to bid more than loggers due to cost differences or the imperfect competition loggers face when selling felled timber to mills.

The median reserve price is $27.77/mbf. Reserve prices in our sample are set according to the "residual value" method, subject to the constraint that 85% of auctions should end in a sale.[14] In our sample 5% of auctions end in no sale.

We define potential entrants as the auction's bidders plus those bidders who bid within 50 km of an auction over the next month. One way of assessing the appropriateness of this definition is that 98% of the bidders in any auction also bid in another auction within 50 km of this auction over the next month. The median number of potential bidders is eight (mean of 8.93) and this is evenly divided between mills and loggers.

We define entrants as the set of bidders we observe at the auction, even if they did not submit a bid above the reserve price.[15] The median number of mill and logger entrants are three and one, respectively. Among the set of potential logger entrants, on average 21.5% enter, whereas on average 66.1% of potential mill entrants enter.

Our model assumes that differences in values explain why mills are more likely than loggers to enter an auction. However, this pattern could also be explained by differences in entry costs, which we allow to vary across auctions, but not across mills and loggers within an

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[13]For estimation we set $\overline{V} = \$500$/mbf, which is substantially above the highest price observed in our sample.

[14]Essentially, the USFS constructed an estimate of the final selling value of the wood and then subtracted logging, transportation, manufacturing and other costs required to generate a marketable product to arrive at a reserve price. The value and cost estimates are based on the government's cruise. See Baldwin, Marshall, and Richard (1997) for a detailed discussion of the method.

[15]This is the definition in Table 1 and we do estimate our model using this definition. However, in our preferred specification, we interpret the data more cautiously and allow bidders that do not submit bids to have entered (paid K), but learned that their value was less than the reserve price.

Variable	Mean	Std. Dev.	25th-tile	50th-tile	75th-tile	N
WINNING BID ($/mbf)	86.01	62.12	38.74	69.36	119.11	847
BID ($/mbf)	74.96	57.68	30.46	58.46	105.01	3426
LOGGER	65.16	52.65	26.49	49.93	90.93	876
MILL	78.36	58.94	32.84	61.67	110.91	2550
LOGGER WINS	0.15	0.36	0	0	0	887
FAIL	0.05	0.21	0	0	0	887
ENTRANTS	3.86	2.35	2	4	5	887
LOGGERS	0.99	1.17	0	1	1	887
MILLS	2.87	1.85	1	3	4	887
POTENTIAL ENTRANTS	8.93	5.13	5	8	13	887
LOGGER	4.60	3.72	2	4	7	887
MILL	4.34	2.57	2	4	6	887
SPECIES HHI	0.54	0.22	0.35	0.50	0.71	887
DENSITY (hundred mbf/acre)	0.21	0.21	0.07	0.15	0.27	887
VOLUME (hundred mbf)	76.26	43.97	43.60	70.01	103.40	887
HOUSING STARTS	1620.80	261.75	1586	1632	1784	887
RESERVE ($/mbf)	37.47	29.51	16.81	27.77	48.98	887
SELL VALUE ($/mbf)	295.52	47.86	260.67	292.87	325.40	887
LOG COSTS ($/mbf)	118.57	29.19	99.57	113.84	133.77	887
MFCT COSTS ($/mbf)	136.88	14.02	127.33	136.14	145.73	887

Table 1: Summary statistics for sample of California ascending auctions from 1982-1989. All monetary figures in 1983 dollars. Here, ENTRANTS are the set of bidders we observe at the auction, even if they did not submit a bid above the reserve price. We count the number of potential entrants as bidders in the auction plus those bidders who bid within 50km of an auction over the next month. SPECIES HHI is the Herfindahl index for wood species concentration on the tract. SELL VALUE, LOG COSTS and MFCT COSTS are USFS estimates of the value of the tract and the logging and manufacturing costs of the tract, respectively. In addition to the USFS data, we add data on (seasonally adjusted, lagged) monthly housing starts, HOUSING STARTS, for each tract's county.

auction. This is unlikely for three reasons. First, there is little reason to believe that the cost of performing cruises differs substantially across firms for any sale since all potential entrants (a) must attend the auction if they want to submit a bid and (b) are interested in similar information when performing their own cruise (even if their values are different). Second, Table 1 clearly shows that mills also bid more than loggers, suggesting that the meaningful distinction between the types are their value distributions. Third, conditional on entering, loggers are still much less likely to win than mills (15.5% vs. 27.9%). This last point argues against the possibility that loggers enter and win less because of high (relative to mills) entry costs, as this would lead them to enter only when they expect to win.

3.1 Evidence of Selection

In this subsection we argue that the data is best explained by a model that allows for potential selection. First, in the type-symmetric mixed strategy equilibrium of a model with non-selective entry (i.e. the LS model) and asymmetric bidder types, whenever the weaker type enters with positive probability, the stronger type enters with probability one. Thus, for any auction with some logger entry, a model with no selection would imply that all potential mills entrants enter. In 54.5% of auctions in which loggers participate, and there are some potential mill entrants, some but not all mills participate. A model with selective entry can rationalize partial entry of both bidder types into the same auction.

Second, a model without selection implies that bidders are a random sample of potential entrants. We can test this by applying a Heckman selection model (Heckman (1976)). This consists of a first stage probit of entry as a function of tract characteristics and a flexible polynomial of potential other mill and logger entrants. Using the predicted probabilities, we form an estimate of the Inverse Mills Ratio and include it in the second stage bid regression. The exclusion restriction is that potential competition affects a bidder's decision to enter an auction, but has no direct effect on values. The results appear in Table 2. Column (1) shows OLS results from regressing all bids on auction covariates and whether the bidder is a logger. Column (2) shows the selection model's results. The positive and significant coefficient on the inverse Mills ratio is consistent with bidders being a positively selected sample of potential entrants. In addition, comparing the coefficient on LOGGER across the columns illustrates that selection masks the difference between logger and mill values.[16] This is expected when only those loggers whose values are likely to be high participate and most mills enter ($S'^*_{\text{mill}} < S'^*_{\text{logger}}$).

The evidence presented in this section strongly suggests that the entry process is selective. However, it does not pin down the degree of selection, much less guarantee that the S model is appropriate. Therefore, we now describe how we estimate our model to measure the degree of selection.

4   Estimation

To take the model to data, we need to specify how the parameters of the model may vary across auctions, as a function of observed auction characteristics and unobserved heterogene-

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[16]An alternative way to test for selection is to regress bids on covariates, including measures of potential entrants. If there is selection, the potential entrants should have a positive impact on submitted bids as the entry threshold will increase. We have run these regressions, using both all bids and the winning bid, and consistently find a positive impact of potential entrants (in logs or levels). The result also holds if we instrument for potential entrants using lagged participation of mills in nearby auctions.

	(1)	(2)
CONSTANT	-5.475***	-5.792***
	(0.849)	(0.852)
LOGGER	-.090***	-.203***
	(0.026)	(0.04)
SCALE SALE	0.003	-.017
	(0.054)	(0.054)
SPECIES HHI	0.025	0.064
	(0.056)	(0.057)
DENSITY	0.016	0.013
	(0.063)	(0.063)
VOLUME	0.0003	0.0002
	(0.0003)	(0.0003)
HOUSING STARTS	0.0002**	0.0002*
	(0.00008)	(0.00008)
log SALE VALUE	2.750***	2.775***
	(0.081)	(0.081)
log LOG COSTS	-1.052***	-1.093***
	(0.066)	(0.067)
log MFCT COSTS	-.262*	-.181
	(0.147)	(0.148)
$\widehat{\lambda}$		0.159***
		(0.044)
R^2	0.4297	0.4319
N	3,426	3,426

Table 2: Selection evidence. In both columns the dependent variable is log of the bid per volume and year dummies are included. Column (2) displays the second stage results of a two step selection model. The first stage probit is of entry where the exogenous shifters are potential other mill and logger entrants, incorporated as a flexible polynomial. $\widehat{\lambda}$ is the estimated inverse Mills ratio from the first stage.

ity. Both types of heterogeneity are likely to be important as the tracts we use differ greatly in observed characteristics, such as sale value, size and wood type, and they also come from different forests so they are likely to differ in other characteristics as well. Both observed and unobserved heterogeneity may affect entry costs and the degree of selection, as well as mean values.

Our estimation approach is based on Ackerberg (2009)'s method of simulated maximum likelihood with importance sampling. This method involves solving a large number of games with different parameters once, calculating the likelihoods of the observed data for each of these games, and then re-weighting these likelihoods during the estimation of the distributions for the structural parameters. This method is attractive when it is believed that the

parameters of the model are heterogeneous across auctions and it would be computationally prohibitive to re-solve the model (possibly many times in order to integrate out over the heterogeneity) each time one of the parameters changes.[17]

To apply the method, we assume that the parameters are distributed across auctions according to the following distributions, where X_a is a vector of observed auction characteristics and $TRN(\mu, \sigma^2, a, b)$ is a truncated normal distribution with parameters μ and σ^2, and upper and lower truncation points a and b.

Location Parameter of Logger Value Distribution: $\mu_{a,\text{logger}} \sim N(X_a\beta_1, \omega^2_{\mu,\text{logger}})$

Difference in Mill/Logger Location Parameters: $\mu_{a,\text{mill}} - \mu_{a,\text{logger}} \sim TRN(X_a\beta_3, \omega^2_{\mu,\text{diff}}, 0, \infty)$

Scale Parameter of Mill and Logger Value Distributions: $\sigma_{Va} \sim TRN(X_a\beta_2, \omega^2_{\sigma_V}, 0.01, \infty)$

α: $\alpha_a \sim TRN(\beta_4, \omega^2_\alpha, 0, 1)$

Entry Costs: $K_a \sim TRN(X_a\beta_5, \omega^2_K, 0, \infty)$

These specifications reflect our assumptions that σ_V, α and K are the same for mills and loggers within any particular auction, even though they may differ across auctions. The lower bound on σ_{Va} is set slightly above zero simply to avoid computational problems that were sometimes encountered when there was almost no dispersion of values.[18] Our estimated specifications also assume that the various parameters are distributed independently across auctions. This assumption could be relaxed, although introducing a full covariance matrix would significantly increase the number of parameters to be estimated and, when we have tried to estimate these parameters, we have not found these coefficients to be consistently significant across specifications. The set of parameters to be estimated are $\Gamma = \{\beta_1, \beta_2, \beta_3, \beta_4, \beta_5, \omega^2_{\mu,\text{logger}}, \omega^2_{\mu,\text{diff}}, \omega^2_{\sigma_V}, \omega^2_\alpha, \omega^2_K\}$, and a particular draw of the parameters $\{\mu_{a,\text{logger}}, \mu_{a,\text{mill}}, \sigma_{Va}, \alpha_a, K_a\}$ is denoted θ.

Denoting the outcome for an observed auction by y_a, the log-likelihood function for a sample of A auctions is

$$\sum_{a=1}^A \log \left(\int L_a(y_a|\theta)\phi(\theta|X_a, \Gamma)d\theta \right) \tag{3}$$

where $L_a(y_a|\theta)$ is the likelihood of the outcome y in auction a given structural parameters θ,

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[17]Hartmann (2006), Hartmann and Nair (forthcoming) and Wang (2010) provide applications of these methods to consumer dynamic discrete choice problems. Bajari, Hong, and Ryan (2010) use a related method to analyze entry into a complete information entry game with no selection.

[18]The problem arises from the fact that numerically solving the game requires calculating expected player profits on a grid of possible values, and numerical problems are encountered when the numerical value calculated for value distribution's pdf is non-zero at only one or two grid points. In practice, when taking draws of the parameters for solving the games, we are also imposing upper truncation points on all of the parameters. We choose large values for these, and varying them has no significant effects on our estimates.

$\phi(\theta|X_a, \Gamma)$ is the pdf of the parameter draw θ given Γ, our distributional assumptions, the unique equilibrium strategies implied by our equilibrium concept and auction characteristics including the number of potential entrants, the reserve price and observed characteristics X_a.

Unfortunately, the integral in (3) is multi-dimensional and cannot be calculated exactly. A natural simulation estimator would be

$$\int L_a(y_a|\theta)\phi(\theta|X_a,\Gamma)d\theta \approx \frac{1}{S}\sum_{s=1}^{S} L_a(y_a|\theta_s) \tag{4}$$

where θ_s is one of S draws from $\phi(\theta|X_a,\Gamma)$. The problem is that this would require us to make new draws of θ_s and re-solve the model S times for each auction in our data each time one of the parameters in Γ changes. Instead we follow Ackerberg by recognizing that

$$\int L_a(y_a|\theta)\phi(\theta|X_a,\Gamma)d\theta = \int L_a(y_a|\theta)\frac{\phi(\theta|X_a,\Gamma)}{g(\theta|X_a)}g(\theta|X_a)d\theta \tag{5}$$

where $g(\theta|X_a)$ is the importance sampling density whose support does not depend on Γ, which is true in our case because the truncation points are not functions of the parameters. This can be simulated using

$$\frac{1}{S}\sum_s L_a(y_a|\theta_s)\frac{\phi(\theta_s|X_a,\Gamma)}{g(\theta_s|X_a)} \tag{6}$$

where θ_s is a draw from $g(\theta|X_a)$. Critically, this means that we can calculate $L_a(y_a|\theta_s)$ for a given set of S draws that do not vary during estimation, and simply change the weights $\frac{\phi(\theta_s|X_a,\Gamma)}{g(\theta_s|X_a)}$, which only involves calculating a pdf when we change the value of Γ rather than re-solving the game.

This simulation estimator will only be accurate if a large number of θ_s draws are in the range where $\phi(\theta_s|X_a,\Gamma)$ is relatively high, and, as is well known, simulated maximum likelihood estimators are only consistent when the number of simulations grows fast enough relative to the sample size. We therefore proceed in two stages. First, we estimate Γ using $S = 2,500$ where $g(\cdot)$ is a multivariate uniform distribution over a large range of parameters which includes all of the parameter values that are plausible. Second, we use these estimates $\widehat{\Gamma}$ to repeat the estimation using a new importance sampling density $g(\theta|X_a) = \phi(\theta_s|X_a, \widehat{\Gamma})$ with $S = 500$ draws per auction. The Appendix provides Monte Carlo evidence that the estimation procedure works well even for smaller values of S.

To apply the estimator, we also need to define the likelihood function $L_a(y_a|\theta)$ based on the data we observe about the auction's outcome, which includes the number of potential entrants of each type, the winning bidder and the highest bids announced during the open

outcry auction by the set of firms that indicated that they were willing to meet the reserve price. A problem that arises when handling data from open outcry auctions is that a bidder's highest announced bid may be below its value, and it is not obvious which mechanism leads to the bids that are announced (Haile and Tamer (2003)).

In our baseline specification we therefore make the following assumptions that we view as conservative interpretations of the information that is in the data: (i) the second highest observed bid (assuming one is observed above the reserve price) is equal to the value of the second-highest bidder[19]; (ii) the winning bidder has a value greater than the second highest bid; (iii) both the winner and the second highest bidder entered and paid K_a; (iv) other firms that indicated that they would meet the reserve price or announced bids entered and paid K_a and had values between the reserve price and the second highest bid; and, (v) all other potential entrants may have entered (paid K_a) and found out that they had values less than the reserve, or they did not enter (did not pay K_a). If a firm wins at the reserve price we assume that the winner's value is above the reserve price. Based on these assumptions, the likelihood of an observed outcome where a type 1 (mill) bidder wins the auction, a type 2 (logger) bidder submits the second highest bid of b_{2a}, and $n_{\tau a} - 1$ other firms of type τ participate (i.e., would pay the reserve or announce bids) out of $N_{\tau a}$ potential entrants would

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[19]Alternative assumptions could be made. For example, we might assume that the second highest bidder has a value equal to the winning bid, or that the second highest bidder's value is some explicit function of his bid and the winning bid. In practice, 96% of second highest bids are within 1% of the high bid, so that any of these alternative assumptions give similar results. We have computed some estimates using the winning bid as the second highest value and the coefficient estimates are indeed similar.

be proportional[20] to the following, where $S_{\tau a}^{\prime*}$ are the equilibrium entry thresholds:

$$
\begin{aligned}
L_a(y|\theta) &\propto f_2(b_{2a}|\theta) * \Pr(enter_2|v_2 = b_{2a}, S_{2a}^{*\prime}, \theta) \times \left(\int_{b_{2a}}^{\overline{V}} f_1(v|\theta) \Pr(enter_1|v_1 = v, S_{1a}^{*\prime}, \theta)dv\right) \\
&\times \left(\int_{R_a}^{b_{2a}} f_1(v|\theta) \Pr(enter_1|v_1 = v, S_{1a}^{*\prime}, \theta)dv\right)^{(n_{1a}-1)} \\
&\times \left(\int_{R_a}^{b_{2a}} f_2(v|\theta) \Pr(enter_2|v_2 = v, S_{2a}^{*\prime}, \theta)dv\right)^{(n_{2a}-1)} \\
&\times \left(1 - \int_{R_a}^{\overline{V}} f_1(v|\theta) \Pr(enter_1|v_1 = v, S_{1a}^{*\prime}, \theta)dv\right)^{(N_{1a}-n_{1a})} \\
&\times \left(1 - \int_{R_a}^{\overline{V}} f_2(v|\theta) \Pr(enter_2|v_2 = v, S_{2a}^{*\prime}, \theta)dv\right)^{(N_{2a}-n_{2a})}
\end{aligned}
\tag{7}
$$

reflecting the contributions to the likelihood of the second highest bidder, the winning bidder, the other firms that attend the auction and those that do not attend, respectively.[21]

4.1 Identification

While we make parametric assumptions to estimate the model, here we informally consider what can identify the parameters of the model.[22] With no unobserved auction heterogeneity in values, it is well known that the distribution of values would be non-parametrically identified if entrants submit their values as bids and there is no selection (Athey and Haile (2002)). The entry process with signals can approach the no selection case, allowing identification of the distribution of values, when the equilibrium signal threshold for entry is very low. The equilibrium entry threshold falls when there are few potential entrants and the reserve price is low, and our data contain considerable variation in reserve prices and the number of potential entrants (see Table 1). When bidders are asymmetric, there may need to be no potential strong-type entrants to generate almost full entry of weak-types.

Once the distribution of values is identified, then the distribution of signal noise and the level of entry costs are identified from the amount of entry and changes in the distribution

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[20]This ignores the binomial coefficients, which do not depend on parameters.

[21]If an entrant wins at the reserve price, then the likelihood is calculated assuming that winning bidder's value is above the reserve.

[22]Marmer, Shneyerov, and Xu (2010) and Einav and Esponda (2008) discuss identification in first price auctions without unobserved heterogeneity.

of observed bids (values) from exogenous variation in S'^*, perhaps due to differences in the reserve price and the number (and type) of potential entrants across auctions. For example, if signals are very precise, then the distribution of values of entrants will be almost perfectly truncated at S'^*. On the other hand, if signals are very imprecise, then the distribution of values of entrants will be more similar to the underlying distribution of values in the population. K will be identified from the probability of entry (higher K will reduce entry) and, because of the zero profit condition, the amount of surplus the marginal entrant can expect in the auction if it enters.

The discussion assumes that firms bid their values and there is no unobserved cross-auction heterogeneity. In our data, auctions operate as open outcry auctions, so it is unreasonable to treat all of the observed bids as values. Moreover, unobserved auction heterogeneity in values may be important. This has lead previous researchers to adopt a parametric estimation approach even if they assume no selection (e.g. Athey, Levin, and Seira (forthcoming) and Krasnokutskaya and Seim (forthcoming)). In addition, we are interested in allowing for unobserved auction heterogeneity in entry costs and signal noise. Due to these issues, it is reasonable to take a fully parametric approach.

5   Results

Table 3 presents the parameter estimates for our structural model. We allow the USFS estimate of sale value and its estimate of logging costs to affect mill and logger values and entry costs since these are consistently the most significant variables in regressions of reserve prices or winning bids on observables, including controls for potential entry, and in the specifications in Table 2. We also control for species concentration since our discussions with industry experts lead us to believe that this matters to firms. We allow for auction-level unobserved heterogeneity (to the econometrician) in all parameters. The rightmost columns show the mean and median values of the structural parameters when we take 10 simulated draws of the parameter for each auction. For the rest of the paper, we refer to these as the "mean" and "median" values of the parameters. All standard errors are based on a non-parametric bootstrap with 100 repetitions.

The coefficients show that tracts with greater sale values and lower costs are more valuable, as one would expect. It does appear that there is both unobserved heterogeneity in values across auctions (the standard deviation of μ_{logger}) and heterogeneity in the difference between mill and logger mean values across auctions (the standard deviation of $\mu_{\text{mill}} - \mu_{\text{logger}}$).

Based on the mean value of the parameters, the mean values of mill and logger potential entrants are \$61.95/mbf and \$42.45/mbf, respectively, a 46% difference. Figure 3 shows the

Parameter	β parameters				ω parameter	Mean	Median
	Constant	log SELL VALUE	log LOG COSTS	SPECIES HHI			
$\mu_{a,\text{logger}}$ $\sim N(X_a\beta_1, \omega^2_{\mu,\text{logger}})$	-9.6936 (1.3690)	3.3925 (0.1911)	-1.2904 (0.1332)	0.2675 (0.1386)	0.3107 (0.0213)	3.5824 (0.0423)	3.5375 (0.0456)
$\mu_{a,\text{mill}} - \mu_{a,\text{logger}}$ $\sim TRN(X_a\beta_3, \omega^2_{\mu,\text{diff}}, 0, \infty)$	3.6637 (0.8890)	-0.4998 (0.1339)	-0.0745 (0.0919)	-0.1827 (0.1007)	0.1255 (0.0163)	0.3783 (0.0242)	0.3755 (0.0249)
σ_{Va} $\sim TRN(X_a\beta_2, \omega^2_{\sigma_V}, 0.01, \infty)$	4.0546 (0.7872)	-0.7379 (0.0994)	0.1393 (0.1025)	0.0895 (0.0813)	0.0796 (0.0188)	0.5763 (0.0273)	0.5770 (0.0302)
α_a $\sim TRN(\beta_4, \omega^2_\alpha, 0, 1)$	0.7127 (0.0509)	-	-	-	0.1837 (0.0446)	0.6890 (0.0362)	0.6992 (0.0381)
K_a $\sim TRN(X_a\beta_5, \omega^2_K, 0, \infty)$	1.9622 (13.2526)	-3.3006 (2.7167)	3.5172 (2.4808)	-1.1876 (1.5721)	2.8354 (0.6865)	2.0543 (0.2817)	1.6750 (0.3277)

Table 3: Simulated maximum likelihood with importance sampling estimates allowing for non-entrants to have paid the entry cost. The rightmost columns show the mean and median values of the structural parameters when we take 10 simulated draws of the parameter for each auction. Standard errors based non-parametric bootstrap with 100 repetitions. $TRN(\mu, \sigma^2, a, b)$ is a truncated normal distribution with parameters μ and σ^2, and upper and lower truncation points a and b. Based on 887 auctions.

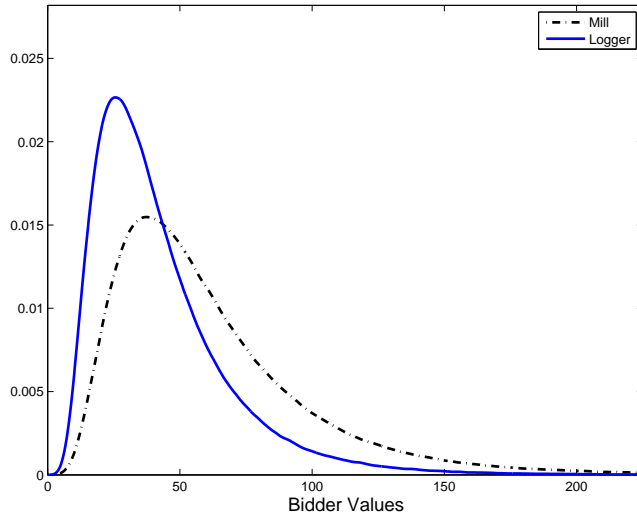value distributions for potential entrants of both types.

Figure 3: Comparing the value distributions for mills (dash-dot) and loggers (solid). Based on the mean value of the parameters from Table 3.

We estimate a mean entry cost of \$2.05/mbf. One forester we spoke with estimated modern day cruising costs of approximately \$6.50/mbf (in 2010 dollars). Converting our estimate to 2010 dollars yields an estimate of \$4.49/mbf. This estimate is consistent with information costs being the majority of entry costs. It is sensible that our estimate is less than the forester's estimate if firms in our data are able to use any information they learn when deciding whether to enter other auctions.

Our estimation approach assumes that, if there are multiple equilibria, the firms will play the equilibrium where mills have the lower S'^*. We checked whether our parameter estimates can support multiple equilibria by plotting type-symmetric "equilibrium best response functions" for mills and loggers for each auction (as we did in Figure 2). For each auction, our parameter estimates support only a single equilibrium. This is because our estimates imply a large difference in the mean values of loggers and mills, relatively low entry costs and a moderate amount of selection (which we discuss in more detail below).

We have also estimated our model under alternative assumptions. For example, we have assumed that there is no unobserved heterogeneity in the parameters across auctions but allowed for all of the parameters to be different across mills and loggers. Under these assumptions it is possible that there may be more than one equilibrium where the type with the higher location parameter for the value distribution has the lower signal threshold for entry. We estimated this model using the iterative nested pseudo-likelihood procedure of Aguirregabiria and Mira (2007). If only one equilibrium is played in the data and the researcher's initial guess of players' beliefs are close enough to those in the data then it can be

hoped - although it is not guaranteed - that this iterative procedure will provide consistent estimates of the parameters even if the underlying model has multiple equilibria. All of the specifications we have tried give the following qualitatively similar conclusions: (i) mills have higher average values than loggers and (ii) the entry procedure is selective for both types, so that, for each type, inframarginal and marginal entrants have value distributions that are clearly different.

Even with the restrictive modeling assumptions we have made, both to deal with the multiple equilibria issue, as well as to take a cautious approach in interpreting the data, the model matches the data fairly well. We slightly over predict logger entry. In the data, on average 0.99 loggers attend each auction. We predict that on average 1.07 loggers will enter and have values greater than the reserve. We slightly under predict mill entry. In the data, on average 2.87 mills attend each auction. We predict that on average 2.44 mills will enter and have values greater than the reserve. We slightly under predict revenues. In the data, the mean (median) revenues are \$81.89/mbf (\$65.89/mbf) and we predict them to be \$75.58/mbf (\$58.13/mbf). We do a good job of matching prices in the event of a sale. In the data, prices average \$85.76/mbf and our model predicts an average of \$86.39/mbf.

The assumptions used to generate our preferred results in Table 3, which we use in all counterfactuals below, are based on a conservative interpretation of what we know about the entry decisions of firms that do not attend the auction. Table 4 shows estimates if we instead assume that firms that did not attend the auction did not pay K. The main differences are slightly (i) more selection and (ii) higher entry costs. This is sensible since we are now assuming that the only firms that enter are those we see and they have values greater than the reserve. A lower α and higher K are needed to rationalize this.

Our estimates of the αs across auctions indicate a moderate amount of selection in the data. Based on our estimates, we find a 46% difference in mean values for potential mill and logger entrants. This is much larger than the average difference in bids across mills and loggers (20.3%), as would be expected if entry is selective. If we consider a representative auction where the reserve and the number of potential mill and logger entrants are set to their respective medians of \$27.77/mbf, four and four, we can compare the difference in values between marginal (those who observed $S_{\tau a}^{\prime *}$) and inframarginal bidders. Based on the mean parameter values, the average mill entrant's value is \$68.13/mbf and the average marginal mill bidder's value is \$45.22/mbf. The fact that the average potential mill entrant's value is higher than the average marginal mill's value reflects the fact that most mills enter. The comparable numbers for entrant and marginal loggers are \$59.80/mbf and \$48.13/mbf, respectively. The difference between marginal and inframarginal bidders is indicative of the degree of selection in the entry process. Also note that, for these estimates, marginal

Parameter	β parameters				ω parameter	Mean	Median
	Constant	log SELL VALUE	log LOG COSTS	SPECIES HHI			
$\mu_{a,\text{logger}}$ $\sim N(X_a\beta_1, \omega^2_{\mu,\text{logger}})$	-8.1176 (1.2141)	3.1520 (0.1922)	-1.3227 (0.1128)	0.1679 (0.1428)	0.2809 (0.0280)	3.5874 (0.0474)	3.5425 (0.0502)
$\mu_{a,\text{mill}} - \mu_{a,\text{logger}}$ $\sim TRN(X_a\beta_3, \omega^2_{\mu,\text{diff}}, 0, \infty)$	2.5003 (0.8217)	-0.3126 (0.1326)	-0.0609 (0.0823)	-0.1337 (0.0929)	0.1503 (0.0154)	0.3709 (0.0280)	0.3668 (0.0290)
σ_{Va} $\sim TRN(X_a\beta_2, \omega^2_{\sigma_V}, 0.01, \infty)$	2.7954 (0.7715)	-0.5725 (0.1117)	0.2009 (0.0942)	0.1466 (0.1004)	0.1307 (0.0216)	0.5807 (0.0268)	0.5802 (0.0277)
α_a $\sim TRN(\beta_4, \omega^2_\alpha, 0, 1)$	0.6094 (0.0321)				0.1343 (0.0300)	0.6086 (0.0312)	0.6072 (0.0316)
K_a $\sim TRN(X_a\beta_5, \omega^2_K, 0, \infty)$	1.7071 (7.0447)	-3.0959 (1.0928)	3.8308 (1.1314)	-1.4711 (0.9802)	2.2488 (0.4595)	2.5308 (0.2637)	2.2353 (0.2688)

Table 4: Simulated maximum likelihood with importance sampling estimates assuming that non-bidders did not pay the entry cost. The rightmost columns show the mean and median values of the structural parameters when we take 10 simulated draws of the parameter for each auction. Standard errors based non-parametric bootstrap with 100 repetitions. $TRN(\mu, \sigma^2, a, b)$ is a truncated normal distribution with parameters μ and σ^2, and upper and lower truncation points a and b. Based on 887 auctions.

loggers tend to have higher values than marginal mills. For illustration, Figure 4 compares the entrants' and marginals' value distributions for each bidder type in the representative auction based on the mean parameters. Due to selection, there is a substantial difference in the marginal and inframarginal bidders for each type.
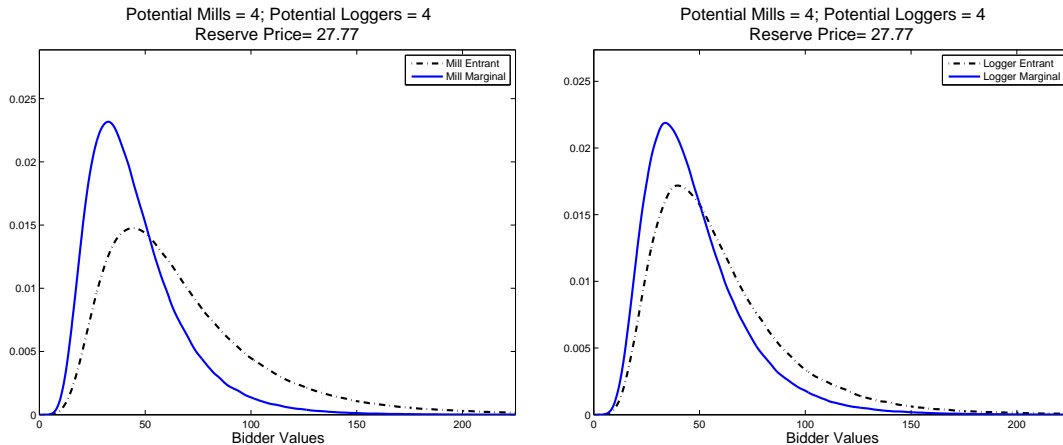

Figure 4: Comparing the value distributions for entrant and "marginal" bidders by type. Based on the mean value of the parameters from Table 3.

Selection directly impacts analysis of any counterfactual that affects the entry margin. An example is the comparison of the USFS's value of adding a marginal and inframarginal bidder to the auction. This is the closest comparison to the experiment in Bulow and Klemperer (1996). They add an additional bidder who has the same value distribution as other bidders. We add another bidder, but consider two cases: one where the additional bidder's value distribution is the same as entrants' and a second where the additional bidder's value distribution is that of a marginal entrant. The second case is the more relevant one since, with endogenous selective entry, it is plausible that additional participants are marginal bidders.

For this, and all counterfactuals below, we define the *seller's value of an auction* as the expected winning bid when the object sells and the seller's value of holding onto the object, v_0, when it does not sell. When considering the representative auction, we assume v_0 is the median reserve price in the data, \$27.77/mbf.[23] When we need to assume a value for v_0 in any particular auction, we use that auction's reserve price. Throughout the paper, a *non-strategic* reserve price means the seller sets a reserve equal to v_0.

Consider the representative auction with four potential mill and four potential logger

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[23]Li and Zheng (2010), and Paarsch (1997), in one of his specifications, also make this assumption. Moreover, in practice this is very similar to assuming that v_0 is the median appraisal value (the USFS's estimated sale value less costs), of \$24/mbf, which is the approach taken by Aradillas-Lopez, Gandhi, and Quint (2010).

entrants. On average 3.18 mills and 1.16 loggers enter and the USFS's expected value of the auction is \$71.48/mbf. If we add a mill bidder whose value is assumed to be distributed as are those of typical mill entrants in this auction, the USFS's value increases by 11.3%. However, it is more likely that if another bidder were added, it would be a "marginal-like" bidder whose signal equaled $S_{\text{mill}}^{*\prime}$. The value of adding this bidder will be much less than adding an "incumbent-like" bidder. If a marginal mill is added, the USFS's value only increases by 4.2%. The respective increases from adding an inframarginal and marginal logger are 8.8% and 5.0%. This comparison is only meant to be illustrative since it assumes that other entrants do not adjust their entry decisions when the additional bidder (marginal or inframarginal) is added. In the next section we analyze the effect of changing potential competition and allow other bidders to adjust their entry decisions accordingly.

6 The Relative Value of Reserve Prices and Competition

In this section we use our structural estimates to show that the USFS's value of setting an optimal reserve price is small relative to increasing competition for its tracts. The reserve price is a natural example of auction design to consider for numerous reasons. First, it is well known that the optimal mechanism for a seller facing a fixed number of symmetric bidders is a second price auction with a correctly chosen reserve price (Myerson (1981) and Riley and Samuelson (1981)). Moreover, analyzing timber auction reserve prices has been an active area of research. Examples of such studies, all of which assume exogenous entry, include Mead, Schniepp, and Watson (1981), Mead, Schniepp, and Watson (1984), Paarsch (1997), Haile and Tamer (2003), Li and Perrigne (2003) and Aradillas-Lopez, Gandhi, and Quint (2010). In addition, government agencies appear to be quite interested in how to set reserve prices in timber auctions and have actively sought the opinions of expert economists (see for example Athey, Cramton, and Ingraham (2003)).

While reserve prices are key to optimal auction design with a given number of bidders, increasing competition may be more valuable to a seller. For example, Bulow and Klemperer (1996) show that sellers prefer auctions with non-strategic reserve prices to those in which they can set an optimal reserve but have one less bidder if (1) bidders are symmetric, (2) all bidders must participate, (3) the marginal revenue curves associated with each bidder are downward sloping and (4) every bidder is willing to make an opening offer of at least v_0.

However, this result does not generally hold in settings with endogenous entry, where the most common way to think about increasing competition is by increasing the number

of potential bidders (e.g. Samuelson (1985), Menezes and Monteiro (2000) or Li and Zheng (2009)). This can be trivially seen in the LS model. In this case, expected revenues decline in the number of potential bidders if bidders participate with probability less than 1, so that a seller dislikes additional potential competition, and prefers the ability to set an optimal reserve price (which is equal to zero for a revenue maximizing seller). Conversely, the seller may prefer to increase the number of potential bidders by one and forgo the opportunity to set a reserve when there is selective entry.[24] In general, the degree of selection affects the seller's returns to both increasing the reserve price and increasing competition because marginal entrants, who will be deterred from entering due to either change, will tend to have lower values than inframarginal entrants.

Table 5 presents results comparing the relative impact of increasing the number of potential entrants and setting an optimal reserve with the original set of potential entrants for the representative auction with $v_0 = \$27.77/\text{mbf}$. With four potential mill and four potential logger bidders, the optimal reserve $R^* = \$55.90/\text{mbf}$ and this increases, relative to setting a non-strategic reserve price, the probability that the auction fails to sell. However, this benefits the seller since he sells at low prices less often and his expected value increases from $\$71.48/\text{mbf}$ to $\$72.14/\text{mbf}$, or 0.92%. Given the informational demands on the seller for setting an optimal reserve (he must know the number of potential entrants, their distribution of values, entry costs, signal distributions and his own value of retaining the object), this improvement is small.

The improvement is also low relative to that based on simply increasing the number of potential entrants by one, regardless of bidder type. Even if the seller continues to set a non-strategic reserve price, his value rises by 6.66% (1.80%) when the number of potential mills (loggers) increases by one: 7.21 (1.96) times the improvement from setting an optimal reserve with the original set of bidders.[25] The benefit of increased competition over optimal reserve pricing is even more pronounced when more than one potential entrant is added.

The returns to both setting an optimal reserve and increasing competition fall in the number of potential bidders. Figure 5 illustrates this effect for the representative auction using the mean parameter estimates. In the figure, the top (bottom) panel shows how changing the number of potential mill (logger) entrants affects the value of adding one potential mill (logger) entrant and setting an optimal reserve price. To understand the figure, consider the top panel (the explanation is analogous for the bottom panel). The dashed line with

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[24]For example, in the symmetric equilibrium of the symmetric bidder-type S model, if values are distributed $logN(5, 0.4)$, $K = 20$ and $N = 4$, with no reserve the seller's expected revenues are 128.9. If instead $N = 3$ but the seller can set the optimal reserve price of 73.9, the seller's expected revenues are 118.17.

[25]This is despite the fact that increasing the number of potential bidders of one type lowers entry probabilities for all other bidders of both types.

N_{Mill}	N_{Logger}	R	USFS's E[Value of Auction]	Pr[No Sale]	E[n_{Mill}]	E[n_{Logger}]	USFS's Gain Relative to 1st Row
4	4	v_0	71.48	0.0016	3.18	1.16	-
4	4	R^*	72.14	0.0706	3.00	1.05	0.92%
4	5	v_0	72.77	0.0015	3.12	1.39	1.80%
4	6	v_0	73.95	0.0014	3.07	1.59	3.46%
5	4	v_0	76.27	0.0011	3.69	0.96	6.66%
6	4	v_0	80.43	0.0009	4.12	0.81	12.51%

Table 5: The relative value of competition for the representative auction based on mean parameters. For any combination of potential bidders and reserve price, the table displays the USFS's expected value of the auction (in \$/mbf), probability of no sale, the expected number of entering mills and loggers and the % increase in the USFS's value relative to having four potential mill and logger entrants and setting a non-strategic reserve price, i.e. the first row in the table. This assumes $v_0 = \$27.77$/mbf. In the second row, $R^* = \$55.90$/mbf. Results are based on the mean parameter estimates in Table 3 and 5,000,000 simulated auctions.

triangles gives the percentage increase in value to the USFS from setting the optimal reserve for the number of potential mill bidders on the horizontal axis and four potential logger bidders, relative to setting a non-strategic reserve when there are that number of mill potential bidders and four potential logger bidders. The dotted line with points gives the percentage increase in value to the USFS from adding one potential mill bidder to the number on the horizontal axis when there are four potential logger bidders, but setting a non-strategic reserve, relative to setting a non-strategic reserve with the original set of potential bidders. The gains are measured on the left axis. The solid line with squares measures the change in value from adding a potential bidder and setting a non-strategic reserve, relative to setting an optimal reserve price with the original set. These relative gains are measured on the right axis. Although the returns to both strategies fall in the number of potential bidders, for this representative auction, the gains from increasing potential competition continue to outweigh those from setting an optimal reserve, and in fact the return of the former, relative to the latter, increases in N.

Since our estimation method recovers structural parameters for each auction, we can evaluate the relative impacts of increasing competition and setting an optimal reserve price for each auction in our data. To do this, we take one draw of the parameters based on the observed characteristics of each auction and assume for each auction that v_0 is the tract's observed reserve price. We then compute the % gain to the USFS from (a) setting an optimal reserve price with that auction's number of potential entrants, (b) setting a non-strategic reserve price but adding one potential mill entrant and (c) setting a non-strategic reserve price but adding one potential logger entrant, all relative to setting a non-strategic reserve price with that auction's number of potential entrants.
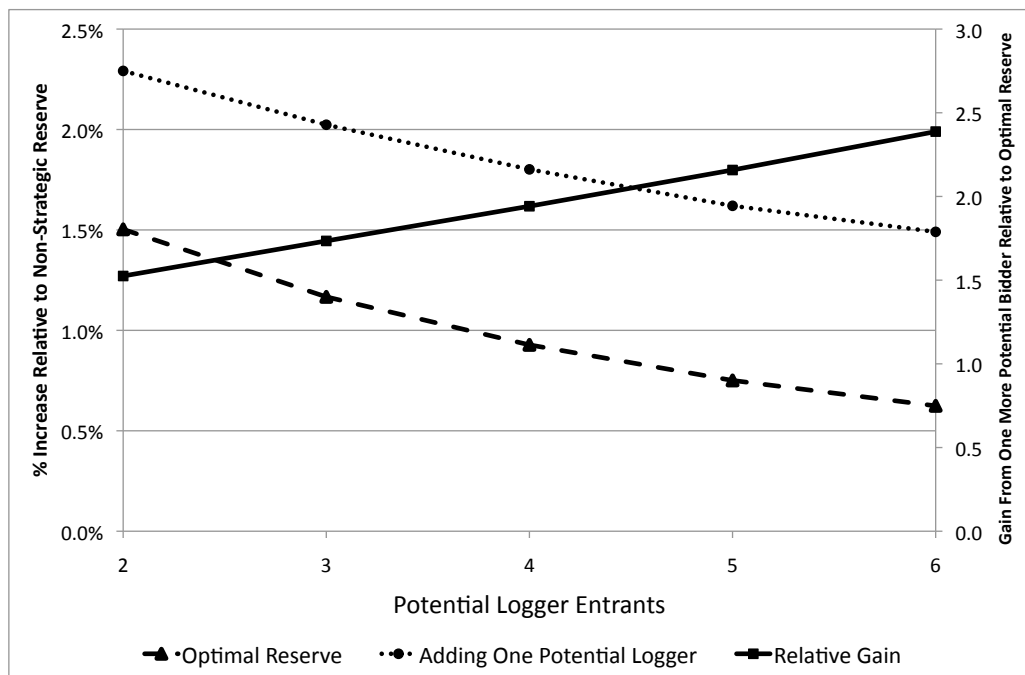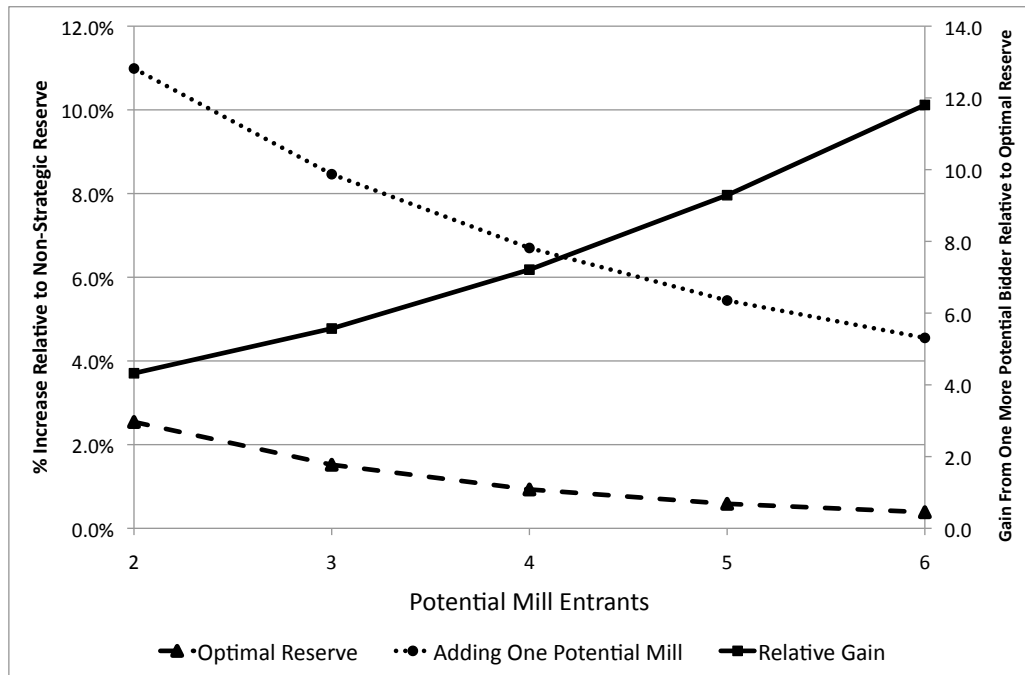
Figure 5: Comparing the effects of adding one extra potential entrant and setting an optimal reserve for different numbers of potential bidders. The top (bottom) panel shows how changing the number of potential mill (logger) entrants affects the value of adding one potential mill (logger) entrant and setting an optimal reserve price. Consider the top panel (the explanation is analogous for the bottom panel). The dashed line with triangles gives the percentage increase in value to the USFS from setting the optimal reserve for the number of potential mill bidders on the horizontal axis and four potential logger bidders relative to setting a non-strategic reserve when there are that number of mill potential bidders and four potential logger bidders. The dotted line with points gives the percentage increase in value to the USFS from adding one potential mill bidder to the number on the horizontal axis when there are four potential logger bidders, but setting a non-strategic reserve, relative to setting a non-strategic reserve with the original set of potential bidders. The gains are measured on the left axis. The solid line with squares measures the change in value from increasing the potential bidders, but setting a non-strategic reserve price, relative to setting an optimal reserve price with the original set of potential bidders.

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Table 6 displays the distributions of the relative returns for (a), (b) and (c). As in the representative auction, the distribution of gains associated with adding one potential mill bidder generally exceeds that for setting the optimal reserve price. For 85.5% of auctions, the USFS prefers adding one potential mill entrant and setting a non-strategic reserve price to setting an optimal reserve price with the original set of potential entrants. The results for an additional logger potential entrant are less clear cut, as the USFS prefers an additional logger for 52.0% of auctions. One reason that an additional logger potential entrant may not always be as valuable to the seller as setting an optimal reserve price is that loggers often have values below our proxy for v_0.[26] As mentioned above, this assumption is necessary for Bulow and Klemper's result that if all bidders must participate, a seller would always prefer an additional bidder to the ability to set an optimal reserve. We can also compare, for each auction, the relative return to adding a potential mill entrant to setting an optimal reserve price (i.e., the return to adding a mill divided by the return to optimal reserve). When we do this, the median value across auctions is 4.94.

There are 38 auctions with one potential bidder, and for these auctions the returns to both setting an optimal reserve price and increasing potential competition tend to be high. In the bottom panel of Table 6 we exclude these cases, and the general pattern that adding competitors is more valuable than setting the optimal reserve price is maintained.

	Percentiles of % Gains Relative to $R = v_0$ with N								
	1	5	10	25	50	75	90	95	99
All Auctions									
R^*	0.00	0.00	0.04	0.21	1.04	4.34	11.31	15.40	59.90
+1 Mill PE	0.09	1.47	2.07	3.20	5.25	9.30	17.40	24.39	83.05
+1 Logger PE	0.00	0.03	0.14	0.54	1.30	3.02	7.17	12.32	59.40
Auctions with > 1 PE									
R^*	0.00	0.01	0.03	0.18	0.93	3.90	9.03	13.09	25.04
+1 Mill PE	0.09	1.51	2.11	3.21	5.13	8.86	15.66	20.80	40.95
+1 Logger PE	0.00	0.03	0.13	0.52	1.26	2.82	6.04	9.75	22.41

Table 6: The distribution of the relative value of competition across sample of auctions. For each auction we compute the % gain to the USFS from (a) setting an optimal reserve price with that auction's number of potential entrants, (b) setting a non-strategic reserve price but adding one potential mill entrant and (c) setting a non-strategic reserve price but adding one potential logger entrant, all relative to setting a non-strategic reserve price with that auction's number of potential entrants. Each row gives percentiles of these gains across auctions and each number is a %. The top panel uses all auctions and the bottom only uses those with more than one potential bidder. Results are based on parameter estimates in Table 3. We assume for each auction that v_0 is the tract's reserve price.

A seller's return from setting an optimal reserve will generally rise in v_0 as this decreases

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[26]For example, our structural estimates imply that across auctions, 38.9% of loggers have values below each auction's observed reserve price (our proxy for v_0).

the cost of not selling. For example, if $v_0 = 0$, which would be the case for a revenue maximizing seller, our conclusions about the relative importance of competition are strengthened. When recalculating the results in Table 6 with $v_0 = 0$ for all auctions, we find that the USFS now prefers an additional mill to setting an optimal reserve price for 99.5% of auctions. It prefers an additional logger for 88.4% of auctions.

On the other hand, it is possible that for high enough v_0, the return from setting an optimal reserve price is greater than that from increasing the number of potential entrants by one. However, the optimal reserve price policy in this case may result in a probability of a failed sale that is too high for some sellers. For example, for the representative auction in our data, the optimal reserve price is more valuable than one additional mill potential entrant only if v_0 is greater than \$57.50/mbf. In this case the probability that there is no sale when the optimal reserve price (\$84.64/mbf) is used is 40%, which greatly exceeds the 15% level that the USFS is allowed.

7   Conclusion

The widespread use of auctions has been cited as an example of the practical relevance of economic theory (Milgrom (2004)), but relatively little is known about the value of the design tools that are the subject of most of the theoretical literature, relative to more fundamental economic factors, such as competition. This is especially true in settings, which are likely to be encountered in the real-world, where there may be multiple bidder types and auction participation may be costly and selective. The degree of selection in the entry process will affect the seller's returns to both increasing the reserve price (the most common design tool in the literature) and increasing competition because marginal entrants, who will be deterred from entering, will tend to have lower values than inframarginal entrants.

We develop and estimate, using data from USFS timber auctions, a flexible entry model for IPV second price and ascending auctions that allows us to evaluate the degree of selection in the entry process. Our empirical model allows for asymmetries between different types of potential bidder and unobserved (to the researcher) auction heterogeneity, which are important characteristics of our data. Our estimates indicate a moderately selective entry process, where, for a representative auction, the average value of a mill (logger) that enters the auction is 50.7% (24.3%) greater than the average value of a marginal mill (logger) entrant. Our estimates also imply the USFS's returns from increased competition, particularly from additional mill (higher value) potential entrants, are much greater than the returns from setting an optimal reserve price. For example, in the representative auction, the USFS's expected value from holding the auction increases by 6.66% when it uses a non-strategic reserve price

but there is one additional mill potential entrant. In contrast, its value increases by only 0.92% when it uses an optimal reserve price without additional competition. This is true even though there has been considerable interest in the literature on optimal reserve prices in timber auctions, based on models with no endogenous entry margin.

We regard our results as contributing to the auction design literature, but the direct relevance of the comparison between reserve prices and competition for organizations or individuals using auctions will obviously depend on how difficult or costly it is to increase the number of buyers that are potentially interested in participating in the auction. It is our view that in many settings there are likely to be numerous ways that a seller, such as the USFS, could encourage additional interest. A simple example is increased advertising for a sale. We also note that it may be costly for the seller to set an optimal reserve price, because this requires precise knowledge of all of the parameters of the model, and, if the optimal reserve price implies a relatively high probability of failed sales, investments by potential bidders that will allow participation in future auctions may also be discouraged.

Our approach can also be used to evaluate the benefit of selection to the seller. For example, given a set of potential entrants, a seller might like to improve the accuracy of bidders' signals, say through more accurate appraisals, if a greater amount of selection improves its value of the auction. In fact, some state forest agencies provide more detailed information about tracts than is provided by the USFS, and the USFS has, subsequent to our data, made more use of outside consultants to improve the quality of its cruises. Looking at the value of this type of policy to sellers, both as a counterfactual and empirically, is one possible direction for future work.

Our results strongly support the view that economic fundamentals can be much more valuable than design tools, as suggested by Klemperer (2002), but we recognize that we rely on some quite common maintained assumptions. First, we assume independent private values. This is a common assumption in the literature on timber auctions (Baldwin, Marshall, and Richard (1997), Brannman and Froeb (2000), Haile (2001) or Athey, Levin, and Seira (forthcoming)) and is made more reasonable by our focus on a sample of auctions where resale possibilities were limited. However, as is well known, common value components could reduce the value of increased potential competition by exacerbating the problem of the "winner's curse" (see Bulow and Klemperer (2002) or Pinkse and Tan (2005)). Second, based on the finding in Athey, Levin, and Seira (forthcoming), we assume that potential bidders do not collude either in their entry decisions or bids. If firms are able to collude, then both design tools and increased competition might help to encourage competitive bidding, and it would also be interesting to understand the relative value of these mechanisms in this case. Third, in this paper we consider only second price or ascending auctions, whereas it has been

suggested in the literature that first-price sealed bid auctions may be effective at increasing the participation of weaker bidders and increasing the seller's revenue when potential bidders are asymmetric (Klemperer (2004) or Athey, Levin, and Seira (forthcoming)). In a related paper (Roberts and Sweeting (2010)), compare revenues from typical simultaneous-bid first and second price auctions with a more sequential bidding structure, and find that, in the presence of selective entry, the gains to using an optimal reserve price in first price auctions are also small.

Our selective entry model can be used to examine many counterfactuals aside from those considered here. With appropriate changes to the second (post-entry) stage of the game, our empirical approach can be used to understand the degree of selection in non-auction settings as well. In the general entry literature, the possibility of selective entry is assumed away. Instead, the different entry decisions by similar firms are explained by i.i.d. differences in fixed costs or entry costs, which do not reflect their subsequent competitiveness or post-entry market outcomes. However, selective entry, which will lead to differences in the competitiveness of marginal and inframarginal entrants, seems intuitively plausible and may affect the conclusions of *any* counterfactual that impacts the set of participating firms.

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A   Monte Carlos: NOT FOR PUBLICATION

This appendix describes a set of Monte Carlo exercises where we investigate the performance of our Simulated Maximum Likelihood (SML) estimator, which uses Importance Sampling, to approximate the likelihood of the observed outcome for a particular auction (Ackerberg (2009)). This evidence is important because SML estimators may perform poorly when the number of simulation draws is too small. We also study the performance of our estimator under alternative definitions of the likelihood, which make different assumptions about the data available to the researcher.

Simulated Data

To generate data for the Monte Carlos, we allow the number of {mill, logger} potential entrants to take on values {3,3}, {5,5}, {8,8}, {6,2} and {2,6} with equal probability. For each auction a, there is one observed auction covariate x_a, which is drawn from a Uniform $[0,1]$ distribution, and the vector X_a is equal to $[1 \ x_a]$. We assume

Location Parameter of Logger Value Distribution: $\mu_{a,\text{logger}} \sim N(X_a\beta_1, \omega^2_{\mu,\text{logger}})$

Difference in Mill/Logger Location Parameters: $\mu_{a,\text{mill}} - \mu_{a,\text{logger}} \sim TRN(X_a\beta_3, \omega^2_{\mu,\text{diff}}, 0, \infty)$

Scale Parameter of Mill and Logger Value Distributions: $\sigma_{Va} \sim TRN(X_a\beta_2, \omega^2_{\sigma_V}, 0.01, \infty)$

$$\alpha: \ \alpha_a \sim TRN(X_a\beta_4, \omega^2_\alpha, 0, 1)$$

$$\text{Entry Costs: } K_a \sim TRN(X_a\beta_5, \omega^2_K, 0, \infty)$$

where $TRN(\mu, \sigma^2, a, b)$ is a truncated normal distribution with parameters μ and σ^2, and upper and lower truncation points a and b. The true values of the parameters are $\beta_1 = [2.8; 1.5]$, $\beta_2 = [0.3; 0.2]$, $\beta_3 = [0.5; -0.1]$, $\beta_4 = [0.5; 0]$, $\beta_5 = [4; 4]$, $\omega_{\mu,\text{logger}} = 0.2$, $\omega_{\sigma_V} = 0.3$, $\omega_{\mu,\text{diff}} = 0.2$, $\omega_\alpha = 0.2$ and $\omega_K = 2$. The reserve price can take on values of 10, 30 or 50. We allow for R to be correlated with x, as one would expect if the seller sets a higher reserve price when he believes the tract has higher value. Specifically, for each auction, we a draw u_a from a uniform $[0,1]$ distribution and set

$$R_a = 10 \text{ if } \frac{x_a + u_a}{2} < 0.33$$

$$R_a = 30 \text{ if } 0.33 \leq \frac{x_a + u_a}{2} \leq 0.66$$

$$R_a = 50 \text{ otherwise.}$$

For each auction we find the unique equilibrium that satisifies the constraint that $S'^*_{\text{mill}} < S'^*_{\text{logger}}$, and generate data using the equilibrium strategies assuming that the auction operates

as a second price sealed bid auction, or, equivalently, an English button auction. The exercises described below all use the same 100 data sets of 1,000 auctions each.

Having constructed the data we estimate the parameters in three different Monte Carlo exercises, which differ in the importance sampling density used to draw the simulated parameters.

A.1 Monte Carlo Exercise 1: Importance Sampling Density is the True Distribution of the Parameters

In the first exercise we make the (generally infeasible) assumption that the researcher knows the true distribution of each of the parameters, which depends on the value of x_a for a particular auction. The number of simulation draws per auction is set equal to 250, and different draws are used for each auction. We compute the results for four different definitions of the likelihood (the same simulation draws are used in each case) that make different assumptions about the information available to the researcher, which will vary with the exact format of the auction (open outcry vs. sealed bid) and with the information that the seller collects about entry decisions. The alternative assumptions are:

1. the researcher observes the values (as bids) and identities of all firms that pay the entry cost and have values above the reserve, and he observes the entry decision of each potential entrant;

2. the researcher observes the values (as bids) and identities of all firms that pay the entry cost and have values above the reserve, and he knows that these firms entered, but for other firms he does not know whether they paid the entry cost and found that their values were less than R, or they did not pay the entry cost;

3. the researcher observes the value and identity of the firm with the second highest value as the final price, the identity of the winning bidder (e.g. whether it is a mill or logger), the identity of all entering firms with values above the reserve price and he observes the entry decision of each potential entrant;

4. the researcher observes the value and identity of the firm with the second highest value as the final price, the identity of the winning bidder (e.g. whether it is a mill or logger), the identity of all entering firms with values above the reserve price, but for other firms he does not know whether they paid the entry cost and found that their values were less than R, or they did not pay the entry cost. This informational assumption forms the basis of the likelihood function shown in equation 7.

Parameter	Variable	True Value	Likelihood Definition			
			1	2	3	4
Logger Location Parameter	Constant	2.8	2.7793 (0.1094)	2.7830 (0.1224)	2.7918 (0.0893)	2.7873 (0.0776)
	x_a	1.5	1.4954 (0.0999)	1.4962 (0.1070)	1.4945 (0.1211)	1.4950 (0.1247)
	Std. Dev.	0.2	0.1904 (0.0182)	0.1930 (0.0204)	0.1849 (0.0320)	0.1848 (0.0312)
Difference in Mill and Logger Location Parameters	Constant	0.3	0.3128 (0.0633)	0.3025 (0.1477)	0.3195 (0.1037)	0.3139 (0.0551)
	x_a	0.2	0.1815 (0.0909)	0.1897 (0.1045)	0.1981 (0.1042)	0.1925 (0.0942)
	Std. Dev.	0.2	0.1873 (0.0190)	0.1888 (0.0278)	0.1872 (0.0286)	0.1820 (0.0277)
Value Distribution Scale Parameter	Constant	0.5	0.5153 (0.1311)	0.5190 (0.1221)	0.5205 (0.1003)	0.5307 (0.0534)
	x_a	-0.1	-0.1007 (0.1099)	-0.1022 (0.1065)	-0.0895 (0.0892)	-0.0795 (0.0759)
	Std. Dev.	0.3	0.2797 (0.0267)	0.2749 (0.0268)	0.2804 (0.0321)	0.2771 (0.0280)
α (Degree of selection)	Constant	0.5	0.4979 (0.1358)	0.4561 (0.2151)	0.4809 (0.1716)	0.5100 (0.0815)
	x_a	0.0	-0.0145 (0.1266)	-0.0403 (0.2003)	-0.0167 (0.1649)	0.0068 (0.1337)
	Std. Dev.	0.2	0.1886 (0.0199)	0.1853 (0.0300)	0.1869 (0.0299)	0.1891 (0.0407)
Entry Cost	Constant	4.0	4.0269 (0.5182)	4.0115 (0.5386)	4.0604 (0.7512)	4.0743 (0.8108)
	K	4.0	4.2420 (0.8683)	4.2858 (0.8868)	4.4381 (1.3409)	4.5171 (1.4110)
	Std. Dev.	2.0	1.9076 (0.2855)	1.9194 (0.3024)	1.9117 (0.3733)	1.8934 (0.3971)

Table 7: True Importance Sampling Density Monte Carlo. The table shows the mean and standard deviation (in parentheses) for each of the parameters estimates across the 100 repetitions based on the four different definitions of the likelihood when we use the true joint distribution of the parameters as the importance sampling density, with $S = 250$ draws. See paper for descriptions of the different likelihood definitions.

Table 7 shows the mean value of each parameter and its standard deviation across the simulated datasets for each definition of the likelihood. With the true distribution as the importance sampling density and $S = 250$, all of the parameters are recovered accurately, including the standard deviation parameters. Several of the parameters appear to be recovered less precisely when less information is available to the researcher (likelihood definition 4), but the differences are never large.

A.2   Monte Carlo Exercise 2: Importance Sampling Density is a Uniform Distribution

When the true distributions are unknown, it is necessary to choose importance sampling densities that provide good coverage of the possible parameter space. In this exercise we draw parameters from independent uniform distributions where $\mu_{a,\text{logger}} \sim U[2,6]$, $\sigma_{Va} \sim U[0.01, 2.01]$, $\mu_{a,\text{mill}} - \mu_{a,\text{logger}} \sim U[0, 1.5]$, $\alpha_a \sim U[0,1]$, $K_a \sim U[0, 20]$. In this case we set the number of simulation draws per auction equal to 1,000 to try to compensate for the fact that a relatively small proportion of the simulated draws are likely to be close to the parameters that really generate the data (in our empirical work we use 2,500 simulated draws per auction so that we get even better coverage). We use the four alternative definitions of the likelihood that we used for the first exercise.

Table 8 shows the mean value of each parameter and its standard deviation across the simulated datasets for each definition of the likelihood. The parameters which determine the means of each distribution are recovered accurately, but four out of the five standard deviation parameters are biased upwards. As in the first exercise, the alternative likelihood definitions appear to have only small effects on the precision of the estimates.

A.3   Monte Carlo Exercise 3: Two Step Estimation

As some of the parameter estimates appear to be biased using a uniform importance sampling density, the estimator we use in the paper uses the estimates based on a uniform importance sampling density to form new importance sampling densities that are used in a repetition of the estimation procedure. As long as the first step estimates are not too biased, this two step procedure should give accurate results, provided that the number of simulation draws is large enough.

To confirm that this is the case, we apply this two step procedure using likelihood definition 4 estimates from exercise 2 for each of the 100 datasets to form an importance sampling density from which we take $S = 250$ simulation draws for each auction (when we apply our

| | | | Likelihood Definition | | | |
Parameter	Variable	True Value	1	2	3	4
Logger Location Parameter	Constant	2.8	2.7051 (0.0969)	2.7102 (0.0998)	2.6895 (0.1324)	2.7031 (0.1333)
	x_a	1.5	1.3921 (0.1766)	1.3807 (0.1810)	1.3331 (0.2090)	1.2946 (0.2245)
	Std. Dev.	0.2	0.2536 (0.0163)	0.2478 (0.0178)	0.2379 (0.0223)	0.2312 (0.0238)
Difference in Mill and Logger Location Parameters	Constant	0.3	0.3286 (0.0806)	0.3255 (0.0848)	0.3532 (0.0970)	0.3407 (0.1007)
	x_a	0.2	0.3073 (0.1465)	0.3148 (0.1505)	0.3536 (0.1595)	0.3819 (0.1713)
	Std. Dev.	0.2	0.2487 (0.0171)	0.2913 (0.0232)	0.2452 (0.0204)	0.2418 (0.0194)
Value Distribution Scale Parameter	Constant	0.5	0.5727 (0.0812)	0.5694 (0.0423)	0.5969 (0.0719)	0.5921 (0.0753)
	x_a	-0.1	-0.0729 (0.0762)	-0.0607 (0.0760)	-0.0476 (0.1227)	-0.0176 (0.1289)
	Std. Dev.	0.3	0.2895 (0.0201)	0.2913 (0.0232)	0.3163 (0.0326)	0.3174 (0.0351)
α (Degree of selection)	Constant	0.5	0.4678. (0.0842)	0.4811 (0.1112)	0.4671 (0.1052)	0.5034 (0.1380)
	x_a	0.0	-0.1070 (0.1526)	-0.1164 (0.1878)	-0.1394 (0.1595)	-0.1849 (0.2112)
	Std. Dev.	0.2	0.2590 (0.0234)	0.3088 (0.0385)	0.2537 (0.0311)	0.3077 (0.0600)
Entry Cost	Constant	4.0	4.3744 (0.7186)	4.0931 (0.7834)	4.7719 (1.0344)	4.6044 (1.0318)
	K	4.0	3.5605 (1.5964)	3.8494 (1.5916)	3.0215 (1.9387)	3.1088 (1.9433)
	Std. Dev.	2.0	3.5409 (0.3358)	3.6859 (0.3446)	3.3704 (0.3623)	3.5099 (0.4164)

Table 8: Uniform Importance Sampling Density Monte Carlo. The table shows the mean and standard deviation (in parentheses) for each of the parameters estimates across the 100 repetitions based on the four different definitions of the likelihood when we use a uniform importance sampling density, with $S = 1,000$ draws. See paper for descriptions of the different likelihood definitions.

estimator to the real data we use $S = 500$). We focus on likelihood definition 4 as it is the basis of our preferred estimates in the paper.

Table 9 shows the mean and standard deviation of the estimates for each of the parameters. We see that both the mean and standard deviation parameters are recovered accurately, although the estimated standard deviation of entry costs is recovered slightly less accurately than when we used the infeasible estimator in exercise 1. Overall, we regard these Monte Carlo results as providing strong support for our estimation procedure, especially as we use more than twice as many simulation draws when we apply our estimator to the actual data.

Parameter	Variable	True Value	Definition 4
Logger	Constant	2.8	2.7313
Location Parameter			(0.1389)
	x_a	1.5	1.3720
			(0.2138)
	Std. Dev.	0.2	0.1722
			(0.0349)
Difference in Mill and Logger	Constant	0.3	0.3308
Location Parameters			(0.0976)
	x_a	0.2	0.3138
			(0.1490)
	Std. Dev.	0.2	0.2039
			(0.0257)
Value Distribution	Constant	0.5	0.5741
Scale Parameter			(0.0639)
	x_a	-0.1	-0.0380
			(0.1078)
	Std. Dev.	0.3	0.2706
			(0.0292)
α (Degree of selection)	Constant	0.5	0.4725
			(0.1321)
	x_a	0.0	-0.0902
			(0.2193)
	Std. Dev.	0.2	0.2064
			(0.0590)
Entry Cost	Constant	4.0	4.2557
			(0.9945)
	K	4.0	3.5161
			(2.0808)
	Std. Dev.	2.0	2.5403
			(0.4681)

Table 9: Two Step Estimator Monte Carlo. The table shows the mean and standard deviation (in parentheses) for each of the parameters estimates across the 100 repetitions based on the fourth different definitions of the likelihood when we use the true joint distribution of the parameters as the importance sampling density, with $S = 250$ draws. See paper for the likelihood definition.